WTW158 2005 Semestertoets 2 Antwoorde

*WTW*158 2005 *Semester test* 2 *Answers*

Beantwoord vrae 1 tot 15 op die MERKLEESVORM se KANT 1

Answer questions 1 *to* 15 *on the OPTIC READER FORM on SIDE* 1

Vraag 1 / *Question* 1

As / *If* $f(x) = \cos^3 x$ dan is / *then* $f'(x) =$

$(1\,a)$ $-3\sin^2 x$	$(1\,b)$ $3\cos^2 x\sin x$	$(1\,c)$ $-3\cos^2 x\sin x$	$(1\,d)$ geen van hierdie nie / *none of these*

Ans: $f'(x) = \frac{d}{dx}\cos^3 x = (3\cos^2 x)(-\sin x) = -3\cos^2 x\sin x$

$(1\,c)$

Vraag 2 / *Question* 2

'n Formule vir $D_x^n(e^{-2x})$ kan die volgende wees / *A formula for* $D_x^n(e^{-2x})$ *appears to be*

$(2\,a)$ e^{-2x}	$(2\,b)$ $-2^n e^{-2x}$	$(2\,c)$ $(-2)^n e^{-2x}$	$(2\,d)$ geen van hierdie nie / *none of these*

Ans: $f(x) = e^{-2x}$, $f'(x) = -2e^{-2x}$, $f''(x) = (-2)(-2)e^{-2x}$, $f'''(x) = (-2)(-2)(-2)e^{-2x}, \ldots$

$\therefore\ f^{(n)}(x) = D_x^n(e^{-2x}) = (-2)^n e^{-2x}$.

$(2\,c)$

Vraag 3 / *Question* 3

Die vergelyking van die raaklyn aan die grafiek van $y = \ln 3x$ in $x = 1$ is

The equation of the tangent line to the graph of $y = \ln 3x$ *at* $x = 1$ *is*

$(3\,a)$ $y = \frac{1}{3}(x-1)$	$(3\,b)$ $y = x-1$	$(3\,c)$ $y - \ln 3 = \frac{1}{3}(x-1)$	$(3\,d)$ $y - \ln 3 = x-1$

Ans: $\frac{d}{dx}(y) = \frac{d}{dx}(\ln 3x) = \frac{3}{3x} = \frac{1}{x}$ $\therefore$ $\frac{dy}{dx} = \frac{1}{x}$

As / *If* $x_1 = 1$ dan is / *then* $y_1 = \ln 3(1) = \ln 3$ en / *and* $m = \frac{dy}{dx}\Big\rfloor_{x=1} = \frac{1}{(1)} = 1$

$\therefore$ die vergelyking van die raaklyn is / $\therefore$ *the equation of the tangent line is*:

$y - y_1 = m(x - x_1) \Rightarrow y - \ln 3 = 1(x-1)$

$(3\,d)$

Vraag 4 / *Question* 4

$\cosh x - \sinh x =$

$(4\,a)$ 1	$(4\,b)$ e^x	$(4\,c)$ $\frac{1}{2}e^{-x}$	$(4\,d)$ e^{-x}

Ans: $\cosh x - \sinh x = \frac{e^x+e^{-x}}{2} - \frac{e^x-e^{-x}}{2} = \frac{e^x+e^{-x}-(e^x-e^{-x})}{2} = e^{-x}$

$(4\,d)$

Vraag 5 / *Question* 5

$\frac{d}{dt}[\cos 3 + e^{t+1} + \ln\sec t] =$

$(5\,a)$ $-\sin 3 + e^{t+1} + \frac{1}{\sec t}$	$(5\,b)$ $e^{t+1} + \tan t$
$(5\,c)$ $-\sin 3 + e^{t+1} + \tan t$	$(5\,d)$ $-\sin 3 + (t+1)e^t + \cos t$

Ans: $\frac{d}{dt}[\cos 3 + e^{t+1} + \ln\sec t] = e^{t+1} + \frac{\sec t\tan t}{\sec t} = e^{t+1} + \tan t$

$(5\,b)$

Vraag 6 / *Question* 6

$\lim\limits_{x\to 5}\frac{2^x-32}{x-5}$ is die afgeleide van / $\lim\limits_{x\to 5}\frac{2^x-32}{x-5}$ *is the derivative of*

$(6\,a)$ $f(x) = 2^x$	$(6\,b)$ $f(x) = 2^x - 32$ in / *at* $x = 5$
$(6\,c)$ $f(x) = 2^x$ in / *at* $x = 2$	$(6\,d)$ geen van hierdie nie / *none of these*

Ans: $\lim\limits_{x \to 5} \frac{2^x-32}{x-5} = \lim\limits_{x \to 5} \frac{(2^x-32)-(0)}{x-5} = \lim\limits_{x \to 5} \frac{(2^x-32)-(2^5-32)}{x-5} = \lim\limits_{x \to 5} \frac{f(x)-f(5)}{x-5} = f'(5)$

$(6\,b)$

Vraag 7 / *Question 7*

Beskou die funksie $f(x) = x - 3\ln x$ op die interval $[1, 4]$. Die globale(absolute) maksimum van f is
Consider the function $f(x) = x - 3\ln x$ *on the interval* $[1, 4]$. *The absolute maximum of* f *is*

$(7\,a)$ ≈ -0.296	$(7\,b)$ ≈ -0.159	$(7\,c)$ 1
$(7\,d)$ geen van hierdie nie / *none of these*		

Ans: $f'(x) = \frac{d}{dx}(x - 3\ln x) = 1 - 3(\frac{1}{x}) = \frac{x-3}{x}$, $f''(x) = \frac{3}{x^2}$
Kritieke getalle / *Critical numbers*: $f'(x) = 0 \Leftrightarrow x - 3 = 0 \Leftrightarrow x = 3$,
$f'(x) \nexists$ as / *if* $x = 0 \notin D_f$
$\therefore$ $x = 3$ is die enigste kritieke getal / $\therefore$ $x = 3$ *is the only critical number*
maar / *but* $f''(3) = \frac{3}{3^2} > 0$
$\therefore$ f het 'n lokale minimum in $x = 3$ nl. $f(3) = (3) - 3\ln(3) \approx -0.29584$
$\therefore$ *f has a local minimum at $x = 3$ i.e.* $f(3) = (3) - 3\ln(3) \approx -0.29584$
Volgens die Ekstreemwaardestelling sal f altyd globale ekstreme hê, want $f(x) = x - 3\ln x$ is kontinu op die geslote interval $[1, 4]$.
According to the Mean Value Theorem f has absolute extremes, since $f(x) = x - 3\ln x$ is continuous on the closed interval $[1, 4]$.
Bereken die funksie-waardes in die eindpunte. / *Calculate the function values at the endpoints.*
$f(1) = (1) - 3\ln(1) = 1$
$f(4) = (4) - 3\ln(4) \approx -0.15888$
$\therefore$ Die globale(absolute) maksimum van f is $f(1) = 1$ / $\therefore$ *The absolute maximum of f is* $f(1) = 1$

$(7\,c)$

Vraag 8 / *Question 8*

Die afgeleide van die funksie $f(x) = \frac{x}{x^2-4}$ is
The derivative of the function $f(x) = \frac{x}{x^2-4}$ *is*

$(8\,a)$ $\frac{1}{2x}$	$(8\,b)$ $\frac{-(x^2+4)}{(x^2-4)^2}$	$(8\,c)$ $\frac{x^2+4}{(x^2-4)^2}$	$(8\,d)$ geen van hierdie nie / *none of these*

Ans: $f'(x) = \frac{d}{dx}\left(\frac{x}{x^2-4}\right) = \frac{1(x^2-4)-x(2x)}{(x^2-4)^2} = \frac{-(x^2+4)}{(x^2-4)^2}$

$(8\,b)$

Vraag 9 / *Question 9*

Laat / *Let* $f(x) = \frac{x}{1+x}$ met / *with* $f'(x) = \frac{1}{(1+x)^2}$.
Die kritieke getal(le) van f is / *The critical number(s) of f is (are)*

$(9\,a)$ -1	$(9\,b)$ 0	$(9\,c)$ 0 en / *and* -1	$(9\,d)$ geen van hierdie nie / *none of these*

Ans: $f'(x) \neq 0$ vir alle / *for all* x.
(of / *or* $f'(x) = \frac{1}{(1+x)^2} = 0 \Leftrightarrow 1 = 0$ onmoontlik / *impossible*)
$f'(x) \nexists$ as / *if* $1 + x = 0 \Leftrightarrow x = -1 \notin D_f$
Geen kritieke getalle / *No critical numbers*

$(9\,d)$

Vraag 10 / *Question 10*

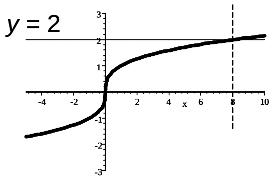
$f(x) = 3x^{\frac{2}{3}} - x$ styg op / $f(x) = 3x^{\frac{2}{3}} - x$ *increases on*

$(10\,a)$ $(-\infty, 0)$	$(10\,b)$ $(0, 8)$	$(10\,c)$ $(8, \infty)$	$(10\,d)$ $(0, \infty)$

Ans: $f'(x) = \frac{d}{dx}\left(3x^{\frac{2}{3}} - x\right) = 3\left(\frac{2}{3}\right)x^{\frac{-1}{3}} - 1 = \frac{2-\sqrt[3]{x}}{\sqrt[3]{x}}$

f styg as / *increases if* $f'(x) > 0 \Leftrightarrow \frac{2-\sqrt[3]{x}}{\sqrt[3]{x}} > 0$

$\times(\sqrt[3]{x})^2$: $(2 - \sqrt[3]{x})\sqrt[3]{x} > 0$



as / *if* $x > 0$ dan moet / *then* $(2 - \sqrt[3]{x}) > 0 \Rightarrow 2 > \sqrt[3]{x} \Rightarrow x < 8$

$\therefore$ $x > 0$ en / *and* $x < 8$ $\qquad\therefore$ $x \in (0, 8)$

as / *if* $x < 0$ dan moet / *then* $(2 - \sqrt[3]{x}) < 0 \Rightarrow 2 < \sqrt[3]{x} \Rightarrow x > 8$

$\therefore$ $x < 0$ en / *and* $x > 8$ $\qquad\therefore$ geen oplossing / *no solution*

$(10\,b)$

Vraag 11 / *Question* 11

Laat / *Let* $f(x) = x + \sqrt{1-x}$ met / *with* $f'(x) = 1 - \frac{1}{2\sqrt{1-x}}$.

In $x = \frac{3}{4}$, het die funksie f / *At* $x = \frac{3}{4}$, *the function* f *has*

$(11\,a)$	'n lokale(relatiewe) minimum / *a local(relative) minimum*
$(11\,b)$	'n lokale(relatiewe) maksimum / *a local(relative) maximum*
$(11\,c)$	'n globale(absolute) minimum / *an absolute minimum*
$(11\,d)$	geen ekstreemwaarde nie / *no extreme value*

Ans: $f'(x) = 1 - \frac{1}{2\sqrt{1-x}} = \frac{2\sqrt{1-x}-1}{2\sqrt{1-x}} = 0 \Leftrightarrow 2\sqrt{1-x} - 1 = 0 \Leftrightarrow \sqrt{1-x} = \frac{1}{2}$

$\Leftrightarrow \left(\sqrt{1-x}\right)^2 = \left(\frac{1}{2}\right)^2 \Leftrightarrow 1 - x = \frac{1}{4} \Leftrightarrow x = \frac{3}{4}$

$\therefore f''(x) = \frac{d}{dx}\left(1 - \frac{1}{2\sqrt{1-x}}\right) = \frac{d}{dx}\left(1 - \frac{1}{2}(1-x)^{\frac{-1}{2}}\right)$

$= \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)(1-x)^{\frac{-3}{2}}(-1) = \frac{-1}{4}(1-x)^{\frac{-3}{2}}$

$\therefore f''\left(\frac{3}{4}\right) = \frac{-1}{4}\left(1 - \frac{3}{4}\right)^{\frac{-3}{2}} = -\left(\frac{1}{4}\right)^{1-\frac{3}{2}} = -2 < 0$

$\therefore$ f het 'n lokale(relatiewe) maksimum / f *has a local(relative) maximum*

$(11\,b)$

Vraag 12 / *Question* 12

Beskou die funksie $f(x) = x + \cos x$ op die interval $[0, 2\pi]$. Dan is f konkaaf na bo op

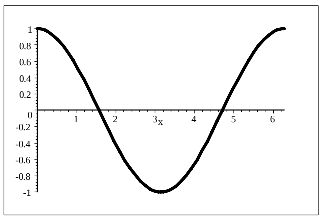
Consider the function $f(x) = x + \cos x$ *on the interval* $[0, 2\pi]$. *Then* f *is concave up(upwards) on*

$(12\,a)$ $[0, \frac{\pi}{2})$ en / *and* $(\frac{\pi}{2}, 2\pi]$	$(12\,b)$ $(\frac{\pi}{2}, 2\pi]$	$(12\,c)$ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
$(12\,d)$ geen interval nie / *no interval*		

Ans: $f'(x) = \frac{d}{dx}(x + \cos x) = 1 - \sin x$

$f''(x) = \frac{d}{dx}(1 - \sin x) = -\cos x$

f is konkaaf na bo as $f''(x) > 0$ / f *is concave up(upwards) if* $f''(x) > 0$



$\Leftrightarrow -\cos x > 0 \Leftrightarrow \cos x < 0 \Leftrightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$(12\ c)$

Vraag 13 / *Question* 13

Die vergelyking(s) van die horisontale asimptoot(asimptote) van $f(x) = \frac{e^x}{1+e^x}$ is
The equation(s) of the horizontal asymptote(s) of $f(x) = \frac{e^x}{1+e^x}$ *is(are)*

$(13\ a)$ $y = -1$ en / *and* $y = 1$	$(13\ b)$ $x = 1$	$(13\ c)$ $y = 0$ en / *and* $y = 1$	$(13\ d)$ $y = 0$

Ans: $\lim_{x\to-\infty} \frac{e^x}{1+e^x} = \frac{0}{1+0} = 0$

$\therefore\ y = 0$ is 'n horisontale asimptoot (links) / $\therefore\ y = 0$ *is a horizontal asymptote (to the left)*

$$\lim_{x\to\infty} \frac{e^x}{1+e^x} \qquad \left(\begin{array}{c}\text{Vorm / Form}\\ \frac{\infty}{\infty}\end{array}\right)$$

$= \lim_{x\to\infty} \frac{e^x}{e^x}$ $\quad$ $(L'H\hat{o}spital)$ $\qquad$ of / *or* $\quad \lim_{x\to\infty} \frac{e^x(1)}{e^x\left(\frac{1}{e^x}+1\right)}$

$= \lim_{x\to\infty} 1 = 1$ $\qquad\qquad$ $= \lim_{x\to\infty} \frac{1}{\frac{1}{e^x}+1} = 1$

$\therefore\ y = 1$ is 'n horisontale asimptoot (regs) / $\therefore\ y = 1$ *is a horizontal asymptote (to the right).*

$(13\ c)$

Vraag 14 / *Question* 14

$\lim_{x\to 0} \frac{\sin x}{\sinh x}$

$(14\ a)$ $= 1$	$(14\ b)$ $= 0$	$(14\ c)$ $= \infty$	$(14\ d)$ bestaan nie / *does not exist*

Ans: $\lim_{x\to 0} \frac{\sin x}{\sinh x} \qquad \left(\begin{array}{c}\text{Vorm / Form}\\ \frac{0}{0}\end{array}\right)$

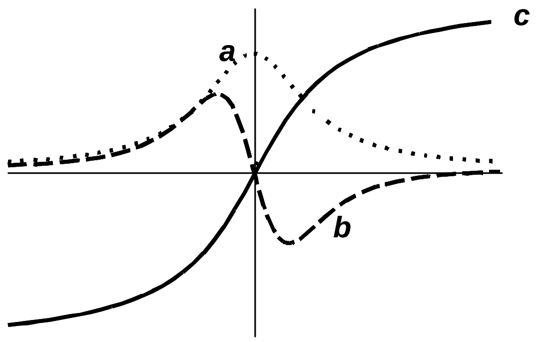
$= \lim_{x\to 0} \frac{\cos x}{\cosh x}$ $\quad$ $(L'H\hat{o}spital)$

$= \frac{1}{1} = 1$

$(14\ a)$

Vraag 15 / *Questions* 15

Beskou die grafieke in die onderstaande skets. Watter van hierdie grafieke stel die funksie f, die eerste afgeleide f' en die tweede afgeleide f'' voor?
Consider the graphs in the sketch below. Which of these graphs represent the function f, *the first derivative* f' *and the second derivative* f''?



$(15\ a)$	a is f, b is f' en / *and* c *is* f''
$(15\ b)$	c is f, a is f' en / *and* b *is* f''
$(15\ c)$	b is f, c is f' en / *and* a *is* f''
$(15\ d)$	b is f, a is f' en / *and* c *is* f''
$(15\ e)$	geen van hierdie / *none of these*

Ans: c is f, a is f' en / *and* b *is* f'',
want $c = f$ styg oral en dit stem ooreen met $a = f' > 0$,
verder as $b = a' = f'' = 0$ het c 'n buigpunt wat ooreenstem met $b = 0$ en $b > 0$ vir $x < 0$
(c konkaaf na bo) en $b < 0$ vir $x > 0$ (c konkaaf na onder).
since $c = f$ *is increasing and that corresponds with* $a = f' > 0$, *also when* $b = a' = f'' = 0$,
c *has an inflection point and that corresponds with* $b = 0$ *and* $b > 0$ *for* $x < 0$
(c *is concave up*) *and* $b < 0$ *for* $x > 0$ (c *is concave down*).

$(15\ b)$

[15]

BEANTWOORD ALLE VERDERE VRAE OP HIERDIE VRAESTEL
en TOON alle bewerkings duidelik aan
ANSWER ALL THE FOLLOWING QUESTIONS ON THE SCRIPT
and SHOW all computations clearly.

Vraag 16 / *Question* 16

Bepaal die eerste afgeleide van elk van die volgende funksies.
Find the first derivative of each of the following functions.

(i) $f(x) = \sec^2 x$

Ans: $f'(x) = \frac{d}{dx}(\sec^2 x) = 2\sec x(\sec x \tan x) = 2\sec^2 x \tan x$

(ii) $y = \sin^{-1}(3x^2)$ (Notasie / *Notation*: $\sin^{-1}x = \arcsin x = \text{bgsin}\, x$)

Ans: $\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1}(3x^2)\right) = \frac{6x}{\sqrt{1-(3x^2)^2}}$

(iii) $f(x) = x^2 \ln x$

Ans: $f'(x) = \frac{d}{dx}(x^2 \ln x) = 2x\ln x + x^2 \times \frac{1}{x} = 2x\ln x + x$

(iv) $f(t) = 10^{2t}$

Ans: $f'(t) = \frac{d}{dt}(10^{2t}) = 10^{2t}(2)(\ln 10) = 2\ln 10 \times 10^{2t}$

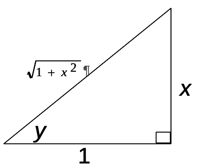
(want as / *since if* $y = 10^{2t}$ dan is / *then* $\ln y = \ln 10^{2t} = 2t\ln 10$
$\therefore \frac{d}{dt}(\ln y) = \frac{d}{dt}(2t\ln 10) \Leftrightarrow \frac{1}{y}\frac{dy}{dt} = 2\ln 10 \quad \therefore \frac{dy}{dt} = y(2\ln 10) = 10^{2t}(2\ln 10) = 2\ln 10 \times 10^{2t}$)

[4]

Vraag 17 / *Question* 17

Bewys dat as $y = \tan^{-1}x$, dan is $\frac{dy}{dx} = \frac{1}{1+x^2}$.
Prove that if $y = \tan^{-1}x$ then $\frac{dy}{dx} = \frac{1}{1+x^2}$.
(Notasie / *Notation*: $\tan^{-1}x = \arctan x = \text{bgtan}\, x$)

Ans: $y = \tan^{-1}x \Leftrightarrow \tan y = x$ en / *and* $x \in \mathbb{R}$ en / *and* $y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x) \Leftrightarrow \sec^2 y \frac{dy}{dx} = 1 \Leftrightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{\tan^2 y + 1} = \frac{1}{x^2+1}$



of uit / *or from* $\therefore \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{\left(\sqrt{1+x^2}\right)^2} = \frac{1}{1+x^2}$

[3]

Vraag 18 / *Question* 18

Laat $f(x) = \sin x$ en neem aan dat $\lim_{x\to 0}\frac{\sin x}{x} = 1$ en $\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$.
Gebruik die definisie van die afgeleide (eerste beginsels) om $f'(x)$ te vind.

Let $f(x) = \sin x$ *and assume that* $\lim_{x\to 0}\frac{\sin x}{x} = 1$ *and* $\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$.
Use the definition of the derivative (first principles) to find $f'(x)$.

Ans: $f'(x) = \lim\limits_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \lim\limits_{h\to 0} \frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$
$= \lim\limits_{h\to 0} \frac{\sin x\cos h-\sin x+\cos x\sin h}{h} = \lim\limits_{h\to 0} \frac{\sin x(\cos h-1)+\cos x\sin h}{h}$
$= \lim\limits_{h\to 0} \frac{\sin x(\cos h-1)}{h} + \lim\limits_{h\to 0} \frac{\cos x\sin h}{h}$ (limietwette / *laws of limits*)
$= \sin x\lim\limits_{h\to 0} \frac{(\cos h-1)}{h} + \cos x\lim\limits_{h\to 0} \frac{\sin h}{h} = \sin x \times 0 + \cos x \times 1$
(want / *since* $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ en / *and* $\lim\limits_{x\to 0} \frac{\cos x-1}{x} = 0$ $\therefore$ $\lim\limits_{h\to 0} \frac{\sin h}{h} = 1$ en / *and* $\lim\limits_{h\to 0} \frac{\cos h-1}{h} = 0$)
$\therefore \frac{d}{dx}\sin x = f'(x) = \cos x$

[3]

Vraag 19 / *Question* 19

Toon aan dat die kromme $y = 6x^3 + 5x - 3$ geen raaklyn met helling 4 het nie.
Show that the curve $y = 6x^3 + 5x - 3$ *has no tangent line with slope* 4.

Ans: $\frac{dy}{dx} = \frac{d}{dx}(6x^3 + 5x - 3) = 18x^2 + 5$
helling van raaklyn / *slope of tangent line*:
$\frac{dy}{dx} = 4 \Leftrightarrow 18x^2 + 5 = 4 \Leftrightarrow 18x^2 = -1$ onmoontlik in $\mathbb{R}$ / *impossible in* $\mathbb{R}$

[2]

Vraag 20 / *Question* 20

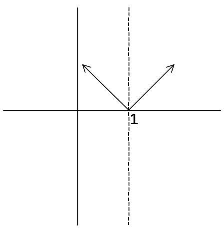
Laat / *Let* $y = g(x)$.
As $y + x\sin y = x^2$ en $g(1) = 0$, gebruik implisiete differensiasie om $g'(1)$ te vind.
If $y + x\sin y = x^2$ *and* $g(1) = 0$, *use implicit differentiation to find* $g'(1)$.

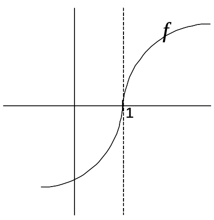
Ans: $\frac{d}{dx}(y + x\sin y) = \frac{d}{dx}(x^2)$
$\Leftrightarrow \frac{dy}{dx} + 1\sin y + x\cos y\,\frac{dy}{dx} = 2x$
$\Leftrightarrow \frac{dy}{dx}(1 + x\cos y) = 2x - \sin y$
$\Leftrightarrow \frac{dy}{dx} = \frac{2x-\sin y}{1+x\cos y}$
$\Rightarrow g'(x) = \frac{2x-\sin g(x)}{1+x\cos g(x)}$ as / *if* $y = g(x)$
$\Rightarrow g'(1) = \frac{2(1)-\sin g(1)}{1+(1)\cos g(1)} = \frac{2-\sin 0}{1+\cos 0} = \frac{2-0}{1+1} = 1$

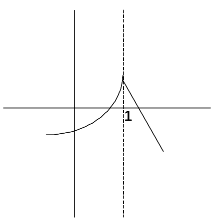
[3]

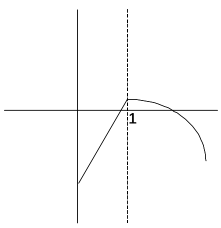
Vraag 21 / *Question* 21

(i) Indien moontlik, skets 'n grafiek van enige funksie wat kontinu is in $x = 1$ maar nie differensieerbaar is in $x = 1$ nie, andersins verduidelik waarom dit onmoontlik is.
If possible, sketch a graph of any function that is continuous at $x = 1$ *but not differentiable at* $x = 1$, *else explain why it is impossible*.

Ans: bv. $y = |x - 1|$ 

of / *or* $y = (x-1)^{\frac{1}{3}}$ 

of / *or* 

of / *or* 

ens., ens. / *etc., etc.*

Enige funksie wat 'n skerppunt of 'n vertikale raaklyn in $x = 1$ het.
Any function that is not smooth, or has a vertical tangent line at $x = 1$..

(ii) Indien moontlik, skets 'n grafiek van enige funksie wat differensieerbaar is in $x = 1$ maar nie kontinu in $x = 1$ is nie, andersins verduidelik waarom dit onmoontlik.
If possible, sketch a graph of any function that is differentiable at $x = 1$ *but not continuous at* $x = 1$, *else explain why it is impossible.*

Ans:
Onmoontlik om so 'n funksie te skets, want volgens 'n stelling ,
as f differensieerbaar is in $x = 1$ is f ook kontinu in $x = 1$ $\big($Stelling 4 p 171$\big)$
Impossible to sketch such a function, since according to a theorem ,
if f is differentiable at $x = 1$ then f is also continuous at $x = 1$ (Theorem 4 p 171)

[3]

Vraag 22 / *Question* 22

Voltooi die Middelwaardestelling: As f kontinu op $[a,b]$ en differensieerbaar op (a,b) is, dan…
Complete the Mean Value Theorem: *If f is continuous on $[a,b]$ and differentiable on (a,b) then…*

Ans: bestaan daar 'n $c \in (a,b)$ sodat $f'(c) = \frac{f(b)-f(a)}{b-a}$.
there exists a $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

[1]

Vraag 23 / *Question* 23

Bepaal $\lim\limits_{x\to 0^+}(\cos x)^{\frac{1}{x}}$ indien dit bestaan. / *Find* $\lim\limits_{x\to 0^+}(\cos x)^{\frac{1}{x}}$ *if it exists.*

Ans: $\lim\limits_{x\to 0^+}(\cos x)^{\frac{1}{x}}$ $\left(\begin{matrix}\text{Vorm / Form}\\ 1^{\infty}\end{matrix}\right)$

Laat / *Let* $y = (\cos x)^{\frac{1}{x}}$ dan is / *then* $\ln y = \ln(\cos x)^{\frac{1}{x}} = \frac{1}{x}\ln(\cos x)$

$$\therefore \lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{1}{x}\ln(\cos x) \quad \left(\begin{matrix}\text{Vorm / Form}\\ \infty\times 0\end{matrix}\right)$$
$$= \lim_{x\to 0^+}\frac{\ln(\cos x)}{x} \quad \left(\begin{matrix}\text{Vorm / Form}\\ \frac{0}{0}\end{matrix}\right)$$
$$= \lim_{x\to 0^+}\frac{\frac{-\sin x}{\cos x}}{1} \quad (L'H\hat{o}spital)$$
$$= \lim_{x\to 0^+}(-\tan x) = 0$$
$$\therefore \lim_{x\to 0^+}y = \lim_{x\to 0^+}e^{\ln y} = e^{\lim\limits_{x\to 0^+}\ln y} = e^0 = 1$$

[3]

Vraag 24 / *Question* 24

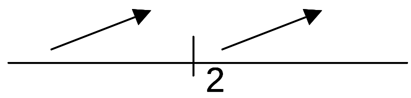
Skets 'n funksie wat aan al die volgende voorwaardes voldoen.
Sketch a function that satisfies all the following conditions.

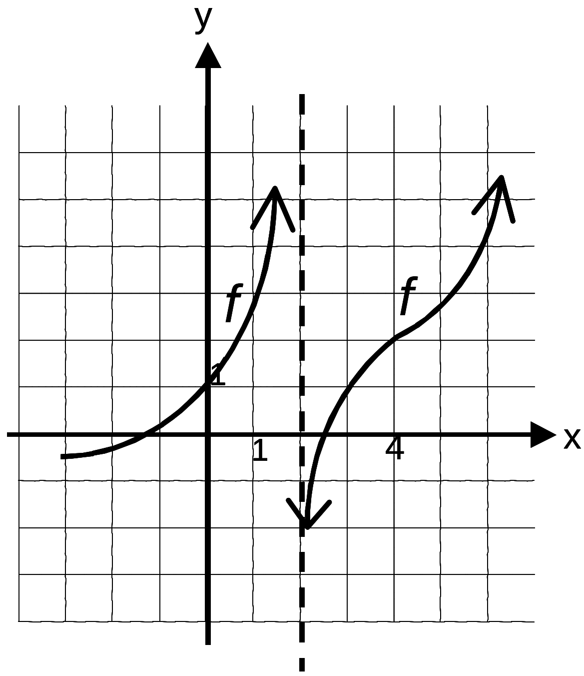
- $f(0) = 1$ en / *and* $f(4) = 2$
- $f'(x) > 0$ vir alle / *for all* $x \neq 2$
- f het 'n vertikale asimptoot in $x = 2$ / f *has a vertical asymptote at* $x = 2$
- $f''(x) > 0$ as / *if* $x < 2$ of / *or* $x > 4$
- $f''(x) < 0$ as / *if* $2 < x < 4$

Ans:

$f'(x) > 0$ vir alle / *for all* $x \neq 2$

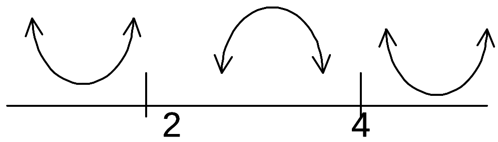
Styg en daal diagram / *Increasing and decreasing diagram*





$f''(x) > 0$ as / *if* $x < 2$ of / *or* $x > 4$	
$f''(x) < 0$ as / *if* $2 < x < 4$	

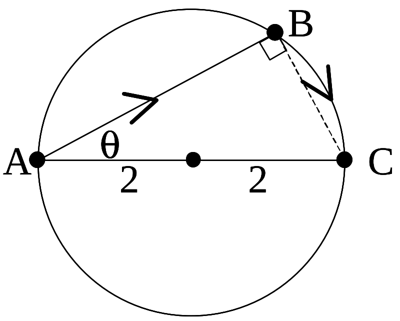
Konkawiteits diagram / *Concavity diagram*



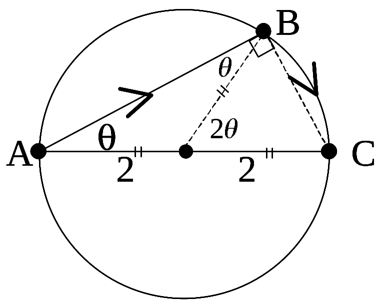
Vraag 25 / *Question* 25

’n Vrou by ’n punt A op die oewer van ’n sirkelvormige meer met radius 2 km. wil aan die anderkant van die meer by ’n punt C lynreg teenoor punt A uitkom. Sy kan stap teen ’n tempo van 4 km per uur en roei met ’n boot teen 2 km per uur.
As sy van punt A tot by punt B roei en dan vanaf punt B na C loop, is die totale afstand afgelê $4\cos\theta + 4\theta$. Op watter wyse moet sy dit aanpak as sy in die kortste tyd moontlik punt C wil bereik?

A woman at a point A on the shore of a circular lake with radius 2 km wants to arrive at the point C diametrically opposite A on the other side of the lake. She can walk at a rate of 4 km per hour and row a boat at 2 km per hour.
If she rows from point A to point B and then walks from point B to C the total distance travelled is $4\cos\theta + 4\theta$. How should she proceed if she wants to reach point C in the shortest possible time?



Ans:



Afstand $= AB + \text{boog}BC = 4\cos\theta + 4\theta$ / *Distance* $= AB + arcBC = 4\cos\theta + 4\theta$

$\therefore \text{Tyd} = \dfrac{AB}{speed\ (roei)} + \dfrac{boogBC}{speed\ (loop)}$ / $Time = \dfrac{AB}{speed\ (row)} + \dfrac{arcBC}{speed\ (walk)}$

Tyd / *Time* = $T(\theta) = \frac{4\cos\theta}{2} + \frac{4\theta}{4} = 2\cos\theta + \theta$
$T'(\theta) = \frac{d}{d\theta}(2\cos\theta + \theta) = -2\sin\theta + 1$
Kritieke getalle / *Critical numbers*:
$T'(\theta) = 0 \Leftrightarrow -2\sin\theta + 1 = 0$
$\Leftrightarrow \sin\theta = \frac{1}{2}$, en / *and* $\theta \in [0, \frac{\pi}{2}]$
$\therefore \theta = \frac{\pi}{6}$
$T''(\frac{\pi}{6}) = -2\cos\frac{\pi}{6} = -\sqrt{3} < 0$
(of / or $T''(\theta) = \frac{d}{d\theta}(-2\sin\theta + 1) = -2\cos\theta \leq 0$ as / *if* $\theta \in [0, \frac{\pi}{2}]$.)
$\therefore T\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) + \left(\frac{\pi}{6}\right) \approx 2.256$ is 'n lokale maksimum / *is a local maximum*
Maar, volgens die Ekstreemwaardestelling sal T altyd globale ekstreme hê, want $T(\theta) = 2\cos\theta + \theta$ is kontinu op die geslote interval $[0, \frac{\pi}{2}]$.
But, according to the Mean Value Theorem T has absolute extremes, since $T(\theta) = 2\cos\theta + \theta$ is continuous on the closed interval $[0, \frac{\pi}{2}]$.
Bereken die funksie-waardes in die eindpunte. / *Calculate the function values at the endpoints.*
$T(0) = 2\cos(0) + (0) = 2$ en / *and* $T\left(\frac{\pi}{2}\right) = 2\cos\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right) = \frac{\pi}{2} \approx 1.571$
$\therefore \frac{\pi}{2} < 2 < 2.256$
Dus sal dit haar die kortste tyd neem om die hele afstand te stap.
Therefore the shortest time will be when she walks all the way.

[4]