2003 (1ste / st semester)
WTW158 Semestertoets 2 Antwoorde / WTW158 Semester test 2 Answers

Vraag 1 / *Question* 1

Q:Bepaal $\frac{dy}{dx}$ as: / *Find* $\frac{dy}{dx}$ *if* :

(i) $\quad y = e^{2x}\cos(3x)$

(ii) $\quad y = x^2 \bullet 2^x$

(iii) $\quad y = \sin^{-1}\left(\sqrt{x+1}\right)$ $\qquad$ (Notasie / *Notation*: $\sin^{-1}x = \arcsin x = \text{bgsin}\,x$)

(iv) $\quad y = \sqrt{4+x^2}$

(v) $\quad \cos(xy) = y$

A: $\quad(i)$ $\quad \frac{dy}{dx} = \frac{d}{dx}(e^{2x}\cos(3x)) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$

A: $\quad(ii)$ $\quad \frac{dy}{dx} = \frac{d}{dx}(x^2 \bullet 2^x) = 2^x(2x) + x^2 2^x \ln 2$

A: $\quad(iii)$ $\quad \frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\left(\sqrt{x+1}\right) = \frac{d}{dx}\arcsin\sqrt{x+1} = \frac{1}{\sqrt{1-\left(\sqrt{x+1}\right)^2}} \times \frac{1}{2}(x+1)^{\frac{-1}{2}} = \frac{1}{2\sqrt{(1+x)}\sqrt{(-x)}}$

A: $\quad(iv)$ $\quad \frac{dy}{dx} = \frac{d}{dx}\sqrt{4+x^2} = \frac{1}{2}(4+x^2)^{\frac{-1}{2}}(2x) = \frac{x}{\sqrt{(4+x^2)}}$

A: $\quad(v)$ $\quad \frac{d}{dx}(\cos(xy)) = \frac{d}{dx}(y)$ $\qquad \therefore -\sin(xy)(y + x\frac{dy}{dx}) = \frac{dy}{dx}$

$\qquad\qquad \therefore (-1 - x\sin(xy))\frac{dy}{dx} = y\sin(xy)$

$\quad \frac{dy}{dx} = \frac{y\sin(xy)}{(-1-x\sin(xy))}$

Vraag 2 / *Question* 2

Vind $f(x)$ as: / *Find* $f(x)$ *if* :

(i) $\quad f'(x) = 4\sin x + 2\cos x$

(i) $\quad f'(x) = x^3\sqrt{x}$

A: $\quad(i)$ $\quad f'(x) = -4(-\sin x) + 2\cos x$ $\qquad \therefore f(x) = -4\cos x + 2\sin x + c$

A: $\quad(ii)$ $\quad f'(x) = x^3\sqrt{x} = x^{\frac{7}{2}}$ $\qquad \therefore f(x) = \frac{2}{9}x^{\frac{9}{2}} + c$

Vraag 3 / *Question* 3

Gebruik die definisie van die afgeleide (eerste beginsels) om aan te toon dat
$D_x(\cos x) = -\sin x$.
Use the definition of the derivative (first principles) to show that $D_x(\cos x) = -\sin x$.

A: $\quad D_x(\cos x) = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0}\frac{\cos x\cosh - \sin x\sinh - \cos x}{h} = \lim_{h\to 0}\frac{\cos x(\cosh - 1) - \sin x\sinh}{h}$

$\quad = \lim_{h\to 0}\cos x\lim_{h\to 0}\frac{(\cosh - 1)}{h} - \lim_{h\to 0}\sin x\lim_{h\to 0}\frac{\sinh}{h} = \cos x \times 0 - \sin x \times 1 = -\sin x$

want / *since* $\quad \lim_{h\to 0}\frac{(\cosh - 1)}{h} = 0$ $\quad$ en / *and* $\quad \lim_{h\to 0}\frac{\sinh}{h} = 1$

Vraag 4 / *Question* 4

Laat / *Let* $f(x) = x + 2\sin x,$ $\quad x \in [0, \pi]$.
Bepaal die vergelyking(s) van horisontale raaklyn(e)aan die grafiek van f.
Find the equation(s) of horizontal tangent line(s) to the graph of f.
$f(x) = x + 2\sin x$

A: $f'(x) = 1 + 2\cos x$

horisontale raaklyn(e) as :/ *horizontal tangent line(s) if:*

$f'(x) = 0$ $\therefore\ 1 + 2\cos x = 0$ $\therefore\ \cos x = \frac{-1}{2}$

$\therefore\ x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ *of/or* $x = \pi + \frac{\pi}{3} \notin [0, \pi]$

$f(\frac{2\pi}{3}) = \frac{2\pi}{3} + 2\sin\frac{2\pi}{3} = \frac{2}{3}\pi + \sqrt{3} \approx 3.8264$

$\therefore\ y = \frac{2}{3}\pi + \sqrt{3} \approx 3.8264$ is die vergelyking v.d. horisontale raaklyn

$\therefore\ y = \frac{2}{3}\pi + \sqrt{3} \approx 3.8264$ *is the equation of the horizontal tangent line.*

Vraag 5 / *Question* 5

Vind die waarde(s) van r waarvoor $y = e^{rx}$ die vergelyking $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 0$ bevredig.

Find the value(s) of r for which $y = e^{rx}$ satisfies the equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 0$.

A: $\frac{d}{dx}y = \frac{d}{dx}e^{rx} = re^{rx}, \frac{d}{dx}(re^{rx}) = r^2 e^{rx}$

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 0$

$\therefore\ r^2 e^{rx} + 5re^{rx} - 6e^{rx} = 0$ $\therefore\ e^{rx}(r^2 + 5r - 6) = 0\ \therefore\ e^{rx}(r+6)(r-1) = 0$

maar / *but* $e^{rx} \neq 0, \forall x$ $\therefore\ (r+6)(r-1) = 0$ $\therefore\ r = -6$ of/ *or* $r = 1$

Vraag 6 / *Question* 6

Laat / *Let* $f(x) = \frac{\ln|x|}{x}$.

(i) Bepaal f se horisontale asimptote (indien enige). Toon alle berekeninge.
        Find the horizontal asymptote(s) of f (if any). Show all computations.

A: $f(x) = \frac{\ln|x|}{x} = \begin{cases} \frac{\ln x}{x} & ,x > 0 \\ \frac{\ln(-x)}{x} & ,x < 0 \end{cases}$

$\lim\limits_{x\to\infty}\frac{\ln|x|}{x} = \lim\limits_{x\to\infty}\frac{\ln x}{x}$ $\left(\begin{array}{c}\text{vorm / form} \\ \frac{\infty}{\infty}\end{array}\right)$ $(L'Hospital)$

$= \lim\limits_{x\to\infty}\frac{\frac{1}{x}}{1} = 0,$

$\lim\limits_{x\to-\infty}\frac{\ln|x|}{x} = \lim\limits_{x\to-\infty}\frac{\ln(-x)}{x}$ $\left(\begin{array}{c}\text{vorm / form} \\ \frac{\infty}{-\infty}\end{array}\right)$ $(L'Hospital)$

$= \lim\limits_{x\to-\infty}\frac{\frac{1}{x}}{1} = 0$

$\therefore\ y = 0$ is die enigste horisontale asimptoot (links en regs)

$\therefore\ y = 0$ *is the only horizontal asymptote.*

(ii) Bepaal f se vertikale asimptote (indien enige). Toon alle berekeninge.
        Find the vertical asymptote(s) of f (if any). Show all computations.

Moontlike vertikale asimptoot waar $f(x)$ ongedefinieer, nl. $x = 0$

Possible vertical asymptote when $f(x)$ is undefined, i.e. $x = 0$

A: $\lim\limits_{x\to 0^+} \dfrac{\ln x}{x}$ $\left(\begin{array}{c}\text{vorm / form}\\[4pt]\frac{-\infty}{+0}\end{array}\right)$
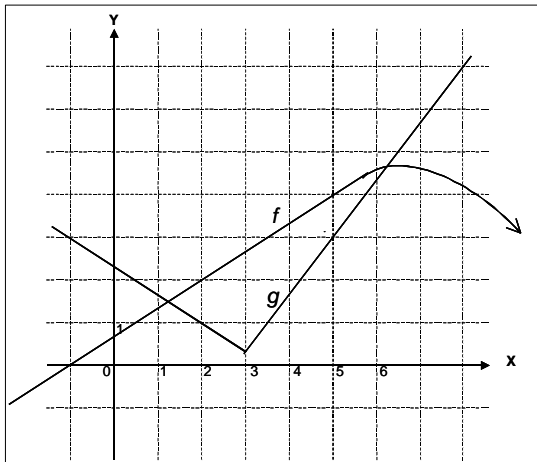
$= -\infty$

of / *or*

(hoef net een vd halwe limiete $+\infty$ of $-\infty$ te kry) / *(need only show that one of the one-sided limits is* $+\infty$ *or* $-\infty$)

$\lim\limits_{x\to 0^-} \dfrac{\ln(-x)}{x}$ $\left(\begin{array}{c}\text{vorm / form}\\[4pt]\frac{-\infty}{-0}\end{array}\right)$

$= \infty$

$\therefore\ x = 0$ is die vertikale asimptoot / *is the vertical asymptote.*

Vraag 7 / *Question 7*

Laat / *Let* $P(x) = f(x)g(x)$, $Q(x) = \dfrac{f(x)}{g(x)}$ en / *and* $C(x) = f(g(x))$.

As f en g die funksies met grafieke soos in die skets, vind:
If f and g are the fuctions whose graphs are shown, find:

A: (i) $P^{\,'}(2)$ $P^{\,'}(x) = f^{\,'}(x)g(x) + f(x)g^{\,'}(x)$
$\therefore P^{\,'}(2) = f^{\,'}(2)g(2) + f(2)g^{\,'}(2) = \frac{4}{6} \times 1 + 2 \times \left(\frac{-2}{3}\right) = -\frac{2}{3}$

A: (ii) $Q^{\,'}(2)$ $Q^{\,'}(x) = \dfrac{f^{\,'}(x)g(x) - f(x)g^{\,'}(x)}{(g(x))^2}$
$\therefore Q^{\,'}(2) = \dfrac{\frac{4}{6}\times 1 - 2\times\frac{-2}{3}}{\left(\frac{-2}{3}\right)^2} = 2$

A: (iii) $C^{\,'}(5)$ $C^{\,'}(x) = f^{\,'}(g(x)) \times g^{\,'}(x)$
$\therefore C^{\,'}(5) = f^{\,'}(g(5)) \times g^{\,'}(5) = f^{\,'}(3) \times \frac{4}{3} = \frac{4}{6} \times \frac{4}{3} = \frac{16}{18} = \frac{8}{9}$

Vraag 8 / *Question 8*

Skets 'n funksie f met die volgende eienskappe :
Sketch a function f with the following properties.

- $\boxed{f \text{ is gedefinieer op } R \ / f \text{ is defined on } R}$

A: $\therefore$ Geen vertikale asimptote of spronge / $\therefore$ *No vertical asymptotes or jumps*

- $\boxed{f'(-1) = 0, \ f'(1) \text{ bestaan nie } / \textit{does not exists}}$

A: $f'(-1) = 0$ $\quad \therefore$ Draaipunt / *Turning point*
A: $f'(1) \ \nexists$ $\quad \therefore$ Skerppunt / *Sharp point*

- $\boxed{f'(x) < 0 \ \text{as} \ / \ \textit{if} \ |x| < 1}$

A: $\therefore f$ is dalend vir $-1 < x < 1$ $\quad \therefore f$ *is decreasing for* $-1 < x < 1$

- $\boxed{f'(x) > 0 \ \text{as} \ / \ \textit{if} \ |x| > 1}$

A: f is stygend vir $x < -1$ of $x > 1$
f *is increasing for* $x < -1$ or $x > 1$

Styg / daal diagram

/ *Increasing/decreasing diagram*

-1   1
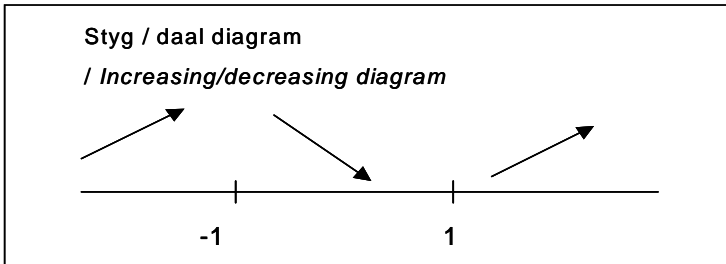
- $\boxed{f(-1) = 4,} \ \boxed{f(1) = 0,} \ \boxed{f''(x) < 0 \ \text{as} \ / \ \textit{if} \ x \neq 1}$

A: f is oral konkaaf na onder (behalwe in $x = 1$)
f *is concave downward everywhere (except in* $x = 1$)

Konkawiteits diagram / *concavity diagram*

1

f '(-1)=0;
f(-1)=4

Y

(-1,4)

4

f

f '(1) does not exists; f(1)=0

-1

(1,0)

X
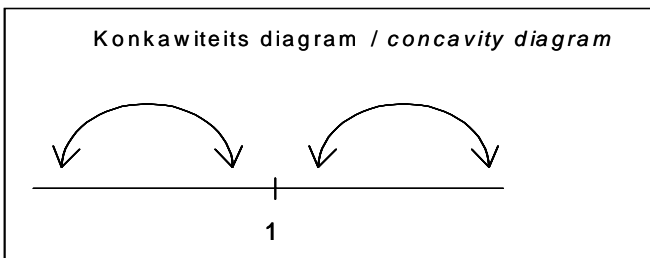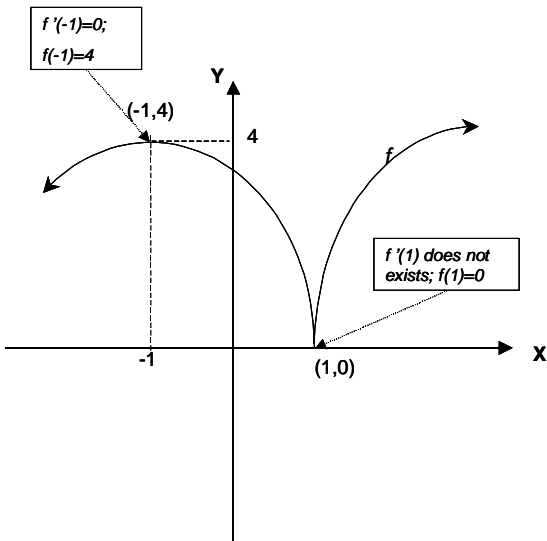
Vraag 9 / *Question* 9

Die grafiek van die AFGELEIDE funksie $f^{\,\prime}$ word gegee.
The graph of the DERIVATIVE function $f^{\,\prime}$ is given.
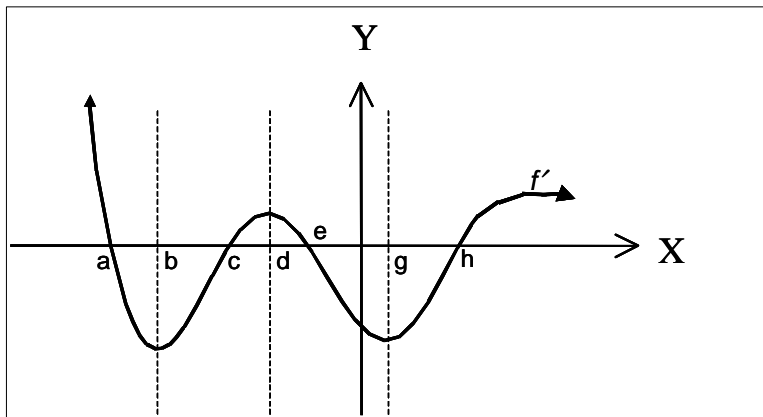
(*i*)   Gee die kritieke getalle van f.
          Give the critical numbers of f.

A: Kritieke getalle van f kom voor as $f^{\,\prime}(x) = 0$ of as $f^{\,\prime}(x) \nexists$
       Critical numbers of f occur when $f^{\,\prime}(x) = 0$ or if $f^{\,\prime}(x) \nexists$.
       $f^{\,\prime}(x) = 0 \Leftrightarrow x = a, x = c, x = e, x = h$ $f^{\,\prime}(x) \nexists$ vir geen waardes van x / *for no values of x*
       $\therefore a; c; e; h$ is die kritieke getalle van f / *is the critical numbers of f.*

(*ii*)   Gee die x-waarde(s) waar daar 'n lokale maksimum is.
           Give the x-value(s) where there is a local maximum.

A: Moontlike maksima (ekstreme) kom slegs voor by die kritieke getalle van f en maksima
wanneer $f^{\,\prime}(x)$ se teken verander van positief na negatief /
       Possible maxima (extremes) only occurs at critical numbers of f and maxima when
       $f^{\,\prime}(x)$ *changes sign from positive to negative.*
$\therefore\; x = a$ en / *and* $x = e$.

(*iii*)   Gee die interval(le) waar f konkaaf na onder is.

Give the interval(s) where f is concave downward.

A: f is konkaaf na onder wanneer $f''(x) < 0$

   f is concave downward when $f''(x) < 0$.

   $\therefore\ x \in (-\infty; b)$ en / *and* $x \in (d; g)$

(iv) Gee die x-waarde(s) waar daar 'n infleksiepunt (buigpunt) is.

   Give the x-value(s) where there is an inflection point.

A: f het 'n <u>moontlike</u> infleksiepunt (buigpunt) waar $f''(x) = 0$ of $f''(x)\ \nexists$.

   $f''(x)$ moet ook verander van teken voordat daar 'n buigpunt is. .

   f has a <u>possible</u> inflection point when $f''(x) = 0$ or $f''(x)\ \nexists$.

   $f''(x)$ should also changes sign before you can say that there is an inflection point.

   $\therefore\ x = b, x = d$ en/*and* $x = g$.