

WTW158 - memo, semester test 1, 2008

Section A: Multiple Choice / Afdeling A: Meervoudige keuse
Time: 45 minutes Marks: 20 / Tyd: 45 minute Punte: 20

The questions in Section A must be completed on SIDE 2 of the optical reader form in SOFT PENCIL.

Die vrae in Afdeling A moet op KANT 2 van die merkleesvorm in SAGTE POTLOOD voltooi word.

Question 1 / Vraag 1

For the piece-wise defined function / Vir die stuksgewys-gedefinieerde funksie

$$f(x) = \begin{cases} 2 & \text{if / as } |x| \leq 1 \\ 3 - x & \text{if / as } |x| > 1 \end{cases}$$

Which statement is true? / Watter bewering is waar?

1a.	The domain of f is $[0, \infty)$ and the range is \mathbb{R} .
1b.	The domain of f is \mathbb{R} and the range is $(-\infty, 2] \cup [4, \infty)$.
1c.	The domain of f is \mathbb{R} and the range is $\mathbb{R} \setminus (2, 4]$.
1d.	The domain of f is $[0, \infty)$ and the range is $(-\infty, 2]$.
1e.	None of the above

1a.	Die definisieversameling van f is $[0, \infty)$ en die waardeversameling van f is \mathbb{R} .
1b.	Die definisieversameling van f is \mathbb{R} en die waardeversameling van f is $(-\infty, 2] \cup [4, \infty)$.
1c.	Die definisieversameling van f is \mathbb{R} en die waardeversameling van f is $\mathbb{R} \setminus (2, 4]$.
1d.	Die definisieversameling van f is $[0, \infty)$ en die waardeversameling van f is $(-\infty, 2]$.
1e.	Geen van bogenoemde

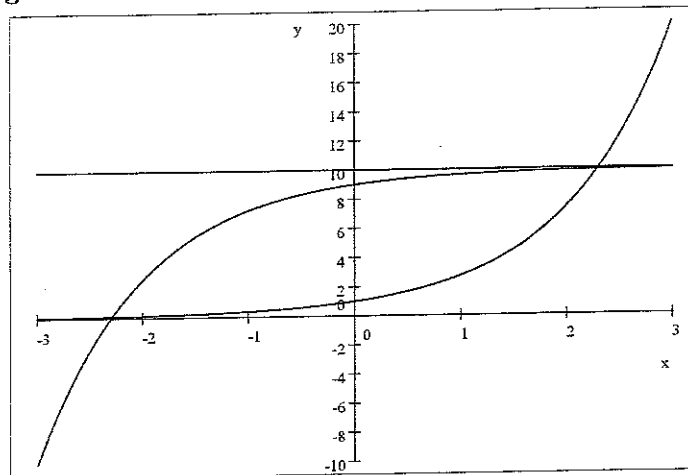
Question 2 / Vraag 2

Given $f(x) = x^3 - 4x$ and $g(x) = 1 + \cos x$. The function $h(x) = \frac{f(x)}{g(x)}$ is

Gegee $f(x) = x^3 - 4x$ en $g(x) = 1 + \cos x$. Die funksie $h(x) = \frac{f(x)}{g(x)}$ is

2a.	Even / Ewe
2b.	Odd / Onewe
2c.	Neither even nor odd / Nie ewe of onewe nie
2d.	Both even and odd / Beide ewe en onewe
2e.	None of the above / Geen van bogenoemde

Question 3 / Vraag 3



The function $f(x) = e^x$ and the transformed function $g(x)$ are shown in the sketch. The function $g(x)$ is given by

Die funksie $f(x) = e^x$ en die getransformeerde funksie $g(x)$ word beide in die skets getoon. Die funksie $g(x)$ word gegee deur

3a.	$g(x) = 10 - e^x$
3b.	$g(x) = e^{-x} + 10$
3c.	$g(x) = -e^x + 10$
3d.	$g(x) = 10 - e^{-x}$
3e.	None of the above / Geen van bogenoemde

Question 4 / Vraag 4

What is the domain of $f(x) = \ln(3 - \sqrt{x-1})$? / Wat is die definisieversameling van $f(x) = \ln(3 - \sqrt{x-1})$?

4a. $0 < x < 4$	4b. $x \geq 1$	4c. $0 < x \leq 1$	4d. $1 \leq x < 10$	4e. None of these / Geen van hierdie
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Question 5 / Vraag 5

$\cos(\arcsin(-\frac{1}{\sqrt{2}})) = \dots\dots\dots$ $\cos(\arcsin(-\frac{1}{\sqrt{2}})) = \dots\dots\dots$

5a. $\frac{5\pi}{4}$	5b. $-\frac{\pi}{4}$	5c. $\frac{1}{\sqrt{2}}$	5d. $-\frac{1}{\sqrt{2}}$	5e. None of these / Geen van hierdie
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Question 6 / Vraag 6

A function f has a domain $[-1, 1]$ and range $[0, 2]$. The inverse g^{-1} of the function $g(x) = f(x - 1) + 1$ has

'n Funksie f het as definisieversameling $[-1, 1]$ en waardeversameling $[0, 2]$. Die inverse g^{-1} van die funksie $g(x) = f(x - 1) + 1$ het

6a.	Domain / Definisieversameling: $[0, 2]$ Range / Waardeversameling: $[1, 3]$
<input checked="" type="radio"/> 6b.	Domain / Definisieversameling: $[1, 3]$ Range / Waardeversameling: $[0, 2]$
6c.	Domain / Definisieversameling: $[1, 3]$ Range / Waardeversameling: $[-2, 0]$
6d.	Domain / Definisieversameling: $[-2, 0]$ Range / Waardeversameling: $[0, 3]$
6e.	None of these / Geen van hierdie

Question 7 / Vraag 7

The function / Die funksie

$$f(x) = \frac{1 + e^{-\frac{1}{x}}}{1 + e^{-x}}$$

<input checked="" type="radio"/> 7a.	has no horizontal asymptotes / het geen horisontale asimptote nie
<input checked="" type="radio"/> 7b.	has one horizontal asymptote $y = 0$ / het een horisontale asimptoot $y = 0$
<input checked="" type="radio"/> 7c.	has one horizontal asymptote $y = 1$ / het een horisontale asimptoot $y = 1$
<input checked="" type="radio"/> 7d.	has one horizontal asymptote $y = 2$ / het een horisontale asimptoot $y = 2$
<input checked="" type="radio"/> 7e.	has two horizontal asymptotes / het twee horisontale asimptote

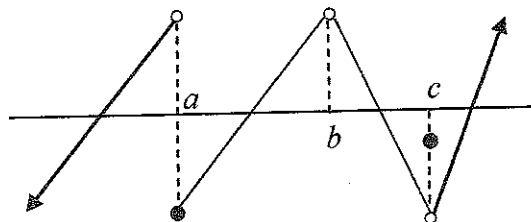
Question 8 / Vraag 8

$$\lim_{x \rightarrow 1^-} \frac{x-1}{|x-1|} = \dots$$

<input checked="" type="radio"/> 8a.	-1	8b.	0	8c.	$-\infty$	8d.	1	8e.	Does not exist / Bestaan nie
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Question 9 / Vraag 9

The graph of f is given / Die grafiek van f word gegee



Which of the following is the statement that is **not true**? / Watter van die volgende is die bewering wat **nie waar is nie**?

9a.	$\lim_{x \rightarrow b} f(x)$ exists / bestaan
9b.	f is right continuous in a / f is regskontinu in a
9c.	$\lim_{x \rightarrow a} f(x)$ does not exist / bestaan nie
9d.	$f(a)$ is undefined / is ongedefinieerd
9e.	$\lim_{x \rightarrow c} f(x) \neq f(c)$

Question 10 / Vraag 10

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$$

Which of the following is the **true** statement? / Watter van die volgende is die bewering wat **waar** is?

10a.	The function could have a jump discontinuity depending on the value of a
10b.	$f(0)$ is undefined
10c.	The function has a removable discontinuity if $a \neq 1$
10d.	The function is continuous for all values of a
10e.	All the statements are true

10a.	Die funksie mag 'n sprong-diskontinuiteit hê, afhangend van die waarde van a .
10b.	$f(0)$ is ongedefinieerd
10c.	Die funksie het 'n verwyderbare diskontinuiteit vir $a \neq 1$
10d.	Die funksie is kontinu vir alle waardes van a
10e.	Al die bewerings is waar.

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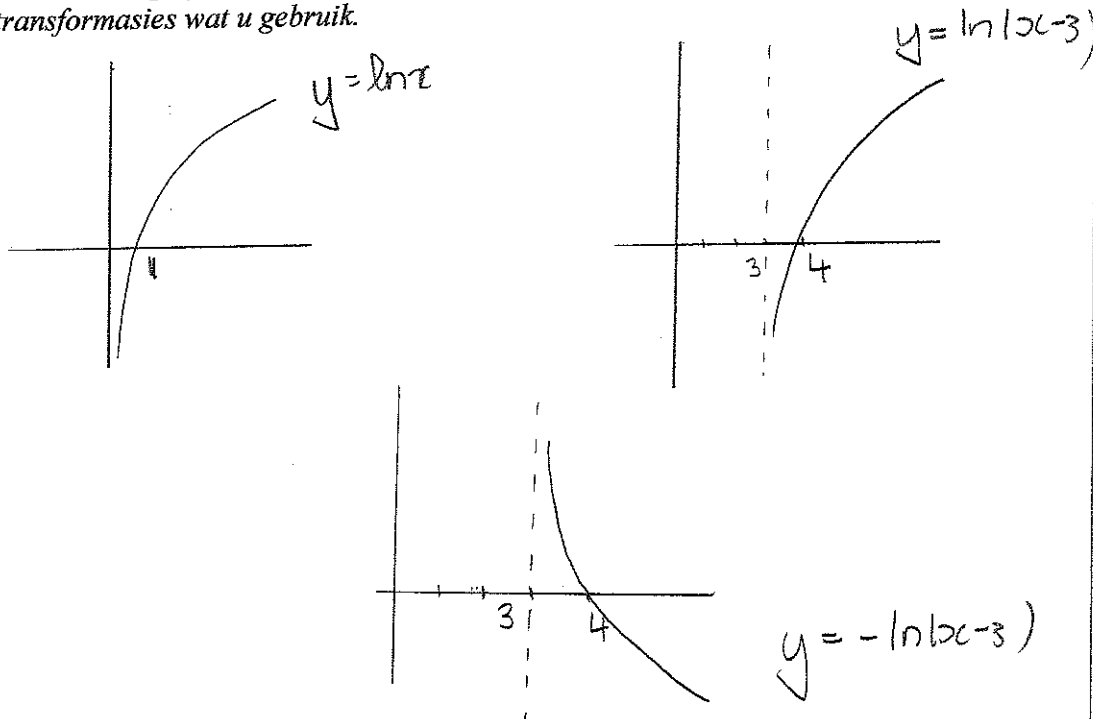
Section B / Afdeling B

Time: 45 min Marks: 20 / Tyd: 45 min Punte: 20

Question 11 / Vraag 11

11.1 Use the graph of $y = \ln x$ to sketch the graph of $y = -\ln(x - 3)$. Show all the transformations that you use.

Gebruik die grafiek van $y = \ln x$ om die grafiek van $y = -\ln(x - 3)$ te skets. Toon al die transformasies wat u gebruik.



[3]

11.2 Solve: / Los op: $-\ln(x - 3) \geq 1$.

$$-\ln(x - 3) \geq 1 \quad \text{and } x > 3$$

$$\Rightarrow \ln(x - 3) \leq -1 \quad \text{and } x > 3$$

$$\Rightarrow e^{\ln(x - 3)} \leq e^{-1} \quad \text{and } x > 3$$

$$\Rightarrow x - 3 \leq e^{-1} \quad \text{and } x > 3$$

[2]

$$\Rightarrow x \leq 3 + e^{-1} \quad \text{and } x > 3$$

$$\Rightarrow x \in (3, 3 + e^{-1}]$$

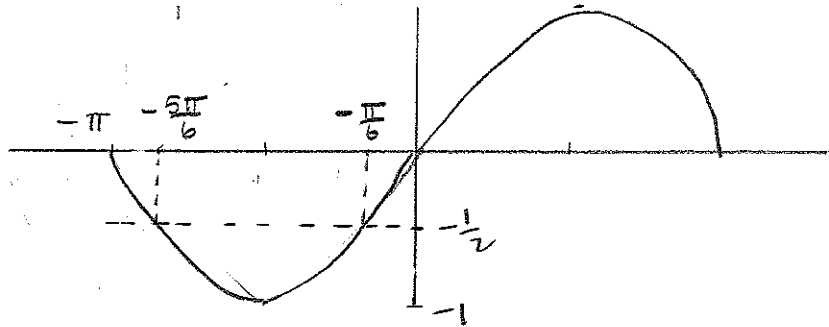
Question 12 / Vraag 12

Use the graph of $f(x) = \sin x$ to solve $\sin x \geq -\frac{1}{2}$, $x \in [-\pi, \pi]$.

Gebruik die grafiek van $f(x) = \sin x$ om $\sin x \geq -\frac{1}{2}$, $x \in [-\pi, \pi]$ op te los.

$$\sin x = -\frac{1}{2} \Rightarrow x = \left(\pi + \frac{\pi}{6}\right) + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{6} + 2k\pi \quad k \in \mathbb{Z}$$

$$= \frac{7\pi}{6} + 2k\pi$$



$$\sin x \geq -\frac{1}{2}, x \in [-\pi, \pi] \Rightarrow x \in \left[-\pi, -\frac{5\pi}{6}\right] \cup \left[-\frac{\pi}{6}, \pi\right]$$

[4]

Question 13 / Vraag 13

13.1 Determine / Bepaal

$$\lim_{x \rightarrow 1} \frac{\sin(x-1)}{2x^2 - x - 1}$$

$$\lim_{x \rightarrow 1} \frac{\sin(x-1)}{(2x^2 - x - 1)}$$

$$= \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x-1)(2x+1)}$$

$$= \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1} \times \lim_{x \rightarrow 1} \frac{1}{2x+1}$$

$$= 1 \times \frac{1}{3}$$

[3]

$$= \frac{1}{3}$$

13.2 Determine / Bepaal

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 3\sqrt{x}}{x^5 - 8x^3 + 10}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2x^2 + 3x^{\frac{1}{2}}}{x^5 - 8x^3 + 10} &= \lim_{x \rightarrow \infty} \frac{\frac{2}{x^3} + \frac{3}{x^{4.5}}}{1 - \frac{8}{x^2} + \frac{10}{x^5}} \\ &= \frac{0 + 0}{1 - 0 + 0} = \frac{0}{1} = 0 \end{aligned}$$

There are other methods

[2]

Question 14 / Vraag 14

Let / Laat

$$f(x) = \begin{cases} \frac{\sqrt{x+4}-2}{x} & \text{if / as } x < 0. \\ 2x + a & \text{if / as } x \geq 0. \end{cases}$$

Determine a such that f is continuous in $x = 0$.

Bepaal a sodat f kontinu is in $x = 0$.

$$f(0) = 2x + a = a$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (2x + a) = a$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \frac{\sqrt{x+4}-2}{x} \quad (\text{form } \frac{0}{0}) \\ &= \lim_{x \rightarrow 0^-} \left(\frac{\sqrt{x+4}-2}{x} \times \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} \right) \end{aligned}$$

$$= \lim_{x \rightarrow 0^-} \frac{x+4-4}{x(\sqrt{x+4}+2)} = \lim_{x \rightarrow 0^-} \frac{1}{\sqrt{x+4}+2} = \frac{1}{4}$$

[3]

$$\lim_{x \rightarrow 0} f(x) \text{ exists if } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) \Rightarrow a = \frac{1}{4}$$

$$f \text{ is continuous in } x=0 \text{ if } \lim_{x \rightarrow 0} f(x) = f(0) \Rightarrow a = \frac{1}{4}$$

Question 15 / Vraag 15

Determine the vertical asymptotes, if any, of / Bepaal die vertikale asymptote, indien enige van

$$f(x) = \frac{|3x-1|}{3x^2+2x-1}$$

$$f(x) = \frac{|3x-1|}{(3x-1)(x+1)} = \begin{cases} \frac{(3x-1)}{(3x-1)(x+1)} & \text{if } 3x-1 > 0 \\ \frac{-(3x-1)}{(3x-1)(x+1)} & \text{if } 3x-1 < 0 \end{cases}$$

[3]

$$= \begin{cases} \frac{1}{x+1} & \text{if } x > \frac{1}{3} \\ \frac{-1}{x+1} & \text{if } x < \frac{1}{3} \end{cases}$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} \frac{-1}{x+1} = -\infty \quad x > -1 \Rightarrow x+1 > 0$$

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{-1}{x+1} = +\infty \quad x < -1 \Rightarrow x+1 < 0$$

$x = -1$ is the only vertical asymptote.