Universiteit van Pretoria/University of Pretoria
Departement Wiskunde en Toegepaste Wiskunde/Department of Mathematics and Applied Mathematics

WISKUNDE/MATHEMATICS WTW158
SEMESTERTOETS 2/SEMESTER TEST 2

Mei/May 2006

Tyd/Time: 90 min

Punte/Marks: Maks/max 40

PUNTE MARKS	27

VAN/SURNAME FOUCHEE

VOORNAME/FIRST NAMES RUDOLPH JOHANNES

STUDENTENOMMER / STUDENT NUMBER: 2 6 1 2 5 2 0 1

HANDTEKENING / SIGNATURE: Fouchee

OMKRING U LESINGGROEPNOMMER IN DIE EERSTE KOLOM
ENCIRCLE YOUR LECTURE GROUP NUMBER IN THE FIRST COLUMN

	Dosent/Lecturer	Lokaal/Venue	Taal/Language
1	Me Mostert	Ing I 1 1-4 & 1-1, Ing II 2-36 & II 2-37	Afrikaans (C,N,S)
2	Me Verwey	Suidsaal	Afrikaans (E,Z,R)
3	Prof Schoeman	Noord-, Suid-, Louwsaal	Afrikaans (B,M,P)
4	Prof Schoeman	Large Chemistry-, Muller-, Te Water Hall	English (E,R,Z)
5	Mr de Beer	Ing I - 1-1	English (C,S)
6	Mr van Wyk	Ing I -1-4, Ing II - 2-37	English (M)
7	Mr Beyers	Ing II - 2-27, Theology 1-9	English (B,N,P)

LEES DIE VOLGENDE INSTRUKSIES

1. Die vraestel bestaan uit bladsye 1 tot 10, asook 'n merkleesvorm. Kontroleer of u vraestel volledig is.
2. **Gebruik kant 2 van die merkleesvorm.**
3. Doen alle krapwerk op die vraestel. Dit word nie nagesien nie.
4. As u nog ruimte vir 'n antwoord nodig het, gebruik enige blanko spasie en dui dit duidelik aan **deur 'n raam daarom te trek.**
5. Werk wat in potlood of rooi ink beantwoord is word nie nagesien nie.
6. U mag nie korrigeer-vloeistof ("Tipp-Ex") gebruik nie.
7. Geen vraestel mag uit die lokaal geneem word nie.
8. Enige navrae oor die nasienwerk moet binne drie dae nadat die toetse teruggegee is, gedoen word.
9. **Toon die nodige stappe en berekeninge.**

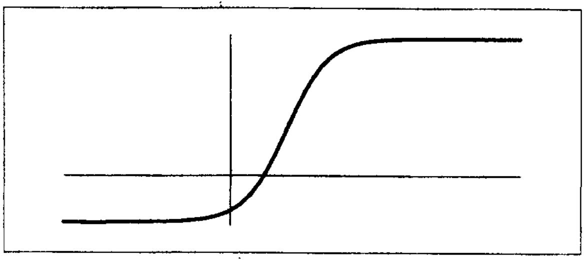
READ THE FOLLOWING INSTRUCTIONS

1. The paper consists of pages 1 to 10, as well as an optic reader form. Check whether your paper is complete.
2. **Use side 2 of the optic reader form.**
3. Do all scribbling on the paper. It will not be marked.
4. If you need more space for an answer, use any blank space and indicate it clearly **by framing it.**
5. Work done in pencil or in red ink will not be marked.
6. You are not allowed to use correction fluid ("Tipp-Ex").
7. No paper may be removed from the venue.
8. Any queries about the marking must be done within three days after the tests have been handed back.
9. **Show the necessary steps and calculations.**

Beantwoord vrae 1 tot 11 op die MERKLEESVORM se KANT 2.
Answer questions 1 to 11 on the OPTIC READER FORM on SIDE 2.

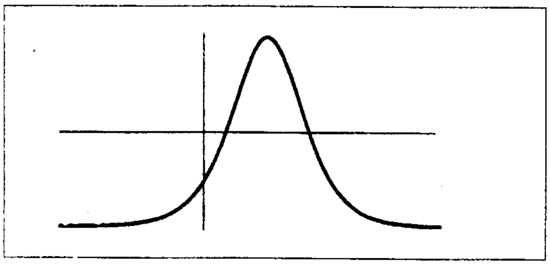
Vraag 1 / Question 1

Beskou die grafiek van f hieronder. / Consider the graph of f below.

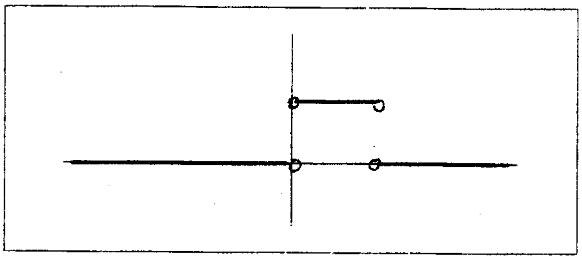


Die grafiek van f' is / The graph of f' is

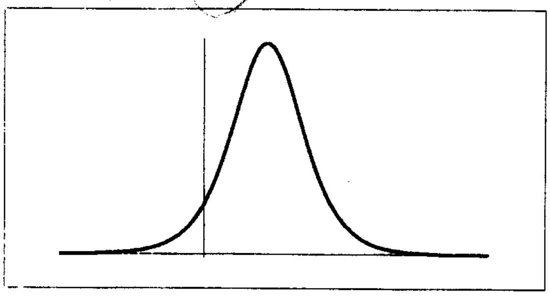
[1a]



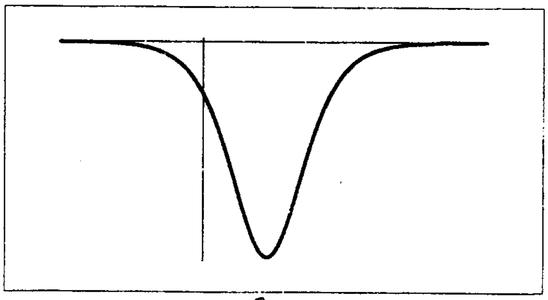
[1b]



[1c]



[1d]



x^2 $2x$

Vraag 2 / Question 2

As f 'n ewe funksie is, dan is f'

If f is an even function, then f' is

[2a] ewe / even	[2b] onewe / odd	[2c] nóg ewe nóg onewe / neither even nor odd

Vraag 3 / Question 3

As $xy = \cot(xy)$, dan is $\frac{dy}{dx} =$

If $xy = \cot(xy)$, then $\frac{dy}{dx} =$

$y + x\frac{dy}{dx} = -\text{cosec}^2(xy)\cdot(y + x\frac{dy}{dx})$

$y + xy' = -\text{cosec}^2(xy)y - \text{cosec}^2(xy)xy'$

$y(1+\text{cosec}^2(xy)) = x\frac{dy}{dx}(-x + \text{cosec}^2(xy)x)$

[3a] $-\frac{y}{x}$	[3b] $\frac{y}{x}$	[3c] $-\text{cosec}^2(xy)[y + x\frac{dy}{dx}]$	[3d] Geen van hierdie / None of these

$\frac{dy}{dx} = \frac{y(1+\text{cosec}^2 xy)}{-x(1+\text{cosec}^2(xy))}$

$= -\frac{y}{x}$

$f = \sin x$
$f' = \cos x$ 1
$f'' = -\sin x$ 2
$f''' = -\cos x$ 3
$f^4 = \sin x$ 4
$f^5 = \cos x$ 5
$f^6 = -\sin x$ 6
$f^7 = -\cos x$ 7
$f^8 = \sin x$ 8

8 16 24 32 40 48 56 64 72

$f^{72} = \sin x$
$f^{73} = \cos x$
$f^{74} = -\sin x$

Vraag 4 / Question 4

As $f(x) = \sin x$, dan is $f^{(74)}(x) =$

If $f(x) = \sin x$, then $f^{(74)}(x) =$

[4a] $\sin x$	[4b] $-\sin x$	[4c] $\cos x$	[4d] $-\cos x$

$1 + \ln x \geq 0$
$\ln x \geq -1$
$x > e^{-1}$

Vraag 5 / Question 5

As $f(x) = \sqrt{1 + \ln x}$, dan is die definisieversameling van f'

If $f(x) = \sqrt{1 + \ln x}$, then the domain of f' is

[5a] $(0, \infty)$	[5b] $[\frac{1}{e}, \infty)$	[5c] $(\frac{1}{e}, \infty)$	[5d] Geen van hierdie / None of these

Vraag 6 / Question 6

$y = x^x$
$\ln y = x \ln x$
$\frac{1}{y} y' = \ln x + \frac{x}{x}$
$y' = x^x(\ln x + 1) + e x^{e-1}$

$\frac{d}{dx} x^e = e x^{e-1}$

As $f(x) = x^x + x^e$, dan is $f'(x) =$

If $f(x) = x^x + x^e$, then $f'(x) =$

[6a] $x\, x^{x-1} + e x^{e-1}$	[6b] $x^x + e x^{e-1}$	[6c] $x^x[1 + \ln x] + e x^{e-1}$	[6d] $x^x[\ln x + e x^{e-1}]$
[6e] Geen van hierdie / None of these			

Vraag 7 / Question 7

$$\frac{d}{dx} \cosh^3 \sqrt{x} = 3 \cosh^2 \sqrt{x} \cdot \sinh \sqrt{x} \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

[7a] $-3\cosh^2 \sqrt{x} . \sinh \sqrt{x} . \frac{1}{2\sqrt{x}}$	[7b] $3\cosh^2 \sqrt{x} . \sinh \sqrt{x}$	[7c] $-\sinh^3 \sqrt{x}$
[7d] $3\cosh^2 \sqrt{x} . \sinh \sqrt{x}\, \frac{1}{2\sqrt{x}}$	[7e] Geen van hierdie / None of these	

Vraag 8 / Question 8

Watter een van die volgende funksies het die afgeleide $\frac{1}{4 + x^2}$?

Which one of the following functions has the derivative $\frac{1}{4 + x^2}$?

[8a] bgtan $\frac{x}{2}$ / arctan $\frac{x}{2}$	[8b] $\frac{1}{2}$ bgtan $\frac{x}{2}$ / $\frac{1}{2}$ arctan $\frac{x}{2}$
[8c] $\frac{1}{4}$ bgtan$x$ / $\frac{1}{4}$ arctan$x$	[8d] $\frac{1}{4}$ bgtan$4x$ / $\frac{1}{4}$ arctan$4x$

$\frac{1}{4} \cdot \frac{1}{1 + 16x}$

Vraag 9 / Question 9

As / If $f(x) = \begin{cases} -1 & \text{as / if } x \leq 0 \\ e^{-\frac{1}{x}} & \text{as / if } x > 0 \end{cases}$, dan is f / then f is

[9a] kontinu en differensieerbaar by 0 / continuous and differentiable at 0
[9b] kontinu maar nie differensieerbaar by 0 nie / continuous but not differentiable at 0
[9c] nie kontinu nie maar differensieerbaar by 0 / not continuous but differentiable at 0
[9d] nie kontinu en nie differensieerbaar by 0 nie / not continuous and not differentiable at 0

Vraag 10 / Question 10

$\lim_{x \to 0} (1 + \frac{k}{x})^x =$

[10a] 0	[10b] 1	[10c] e^k	[10d] $\frac{1}{e^k}$	[10e] Geen van hierdie / None of these

Vraag 11 / Question 11

$\lim_{x \to 0^+} \frac{\ln x}{x} =$ $\lim_{x \to 0}$ $\frac{1}{x}$

[11a] ∞	[11b] $-\infty$	[11c] 1	[11d] bestaan nie / does not exist

[11]

BEANTWOORD ALLE VERDERE VRAE OP HIERDIE VRAESTEL. TOON ALLE BEREKENINGE DUIDELIK.

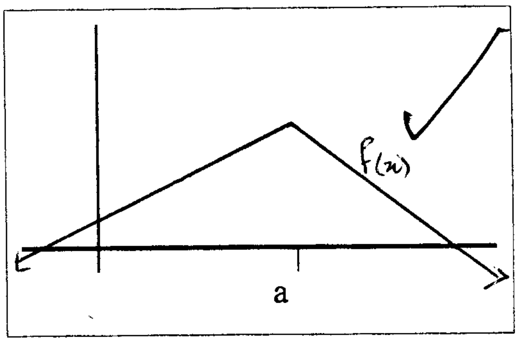
ANSWER ALL THE FOLLOWING QUESTIONS ON THIS PAPER. SHOW ALL COMPUTATIONS CLEARLY.

Vraag 12 / Question 12

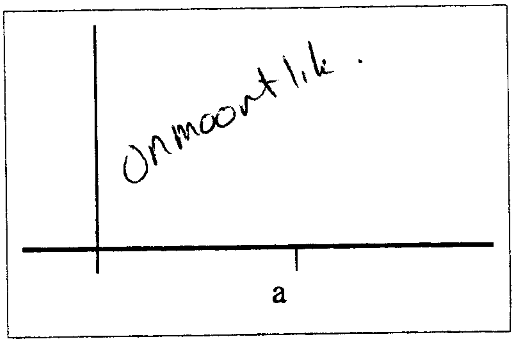
Indien moontlik, skets 'n funksie f met die gegewe eienskap. Indien dit nie moontlik is nie, verduidelik waarom nie.
If possible, sketch a function f with the given property. If not possible, explain why not.

i f is kontinu maar nie differensieerbaar by $x = a$ nie.
f is continuous but not differentiable at $x = a$.

ii f is differensieerbaar by $x = a$, maar nie kontinu by $x = a$ nie.
f is differentiable at $x = a$, but not continuous at $x = a$.



i



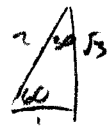
ii

Vir 'n funksie om differensieerbaar te wees, moet die funksie kontinu wees.

[2]

Vraag 13 / Question 13

i Voltooi: / Complete: $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x} = 0$



1

[1]

ii Gebruik eerste beginsels om die helling van die raaklyn aan $f(x) = \sin x$ by $x = \frac{\pi}{3}$ te bepaal.

Use first principles to determine the gradient of the tangent to $f(x) = \sin x$ at $x = \frac{\pi}{3}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \sin x . 0 + \cos x . 1$$

aangesien $\lim\limits_{h \to 0} \dfrac{\sin h}{h} = 1$ en $\lim\limits_{h \to 0} \dfrac{\cos h - 1}{h} = 0$

$\therefore f'(x) = \cos x$

$\therefore$ Helling van raaklyn by $x = \frac{\pi}{3}$ is $f'(\frac{\pi}{3})$

$$f'(\tfrac{\pi}{3}) = \cos \frac{\pi}{3}$$

$$= \frac{1}{2}$$

4

[4]

iii Bepaal die vergelyking van die raaklyn aan $f(x) = \sin x$ by $x = \frac{\pi}{3}$

Determine the equation of the tangent line to $f(x) = \sin x$ at $x = \frac{\pi}{3}$

$$f\left(\frac{\pi}{3}\right) = \sin \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

[1]

Raaklyn: $y - y_1 = m(x - x_1)$

$$y - \frac{\sqrt{3}}{2} = \frac{1}{2}\left(x - \frac{\pi}{3}\right)$$

$$y = \frac{x}{2} - \frac{\pi}{6} + \frac{3\sqrt{3}}{6}$$

1

$$y = \frac{x}{2} - \frac{\pi + 3\sqrt{3}}{6}$$

Vraag 14 / Question 14

Bepaal die afgeleides van die volgende funksies:
(MOENIE JOU ANTWOORDE VEREENVOUDIG NIE!)

Determine the derivatives of the following functions:
(DO NOT SIMPLIFY YOUR ANSWERS!)

i $f(x) = (\text{bgsin}(2x))^{-3}$ / $f(x) = (\arcsin(2x))^{-3}$

$= -3(\text{bgsin}(2x))^{-4} \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$

ii $f(x) = 2^x \sin(mx)$

$= 2^x \cdot \ln 2 \cdot \sin(mx) + 2^x \cdot \cos(mx) \cdot m$

iii $f(x) = \cos\pi$

$= 0$

iv $f(x) = e^{\sinh x^2}$

$= e^{\sinh x^2} \cdot \cosh x^2 \cdot 2x$

v $f(x) = \ln(x \ln x)$

$= \frac{1}{x \ln x} \cdot \left(1 \cdot \ln x + \frac{x}{x}\right)$

vi $f(x) = \frac{\tan x}{(x^2+4)^4}$

$$= \frac{\sec^2 x \cdot (x^2+4)^4 - \tan x\left[4(x^2+4)^3 \cdot 2x\right]}{\left[(x^2+4)^4\right]^2}$$

6

[6]

Vraag 15 / Question 15

Laat / Let $f(x) = \frac{\ln x}{x}$, $x \in [1,3]$.

i Verduidelik waarom f absolute ekstreemwaardes (globale maksimum en globale minimum) op die interval het.

Explain why f has absolute extreme values (global maximum and global minimum) on the interval.

~~f het absolute ekstreemwaardes op die interval want daar bestaan 'n getal c en d in die interval sodanig dat $f(c) \leq f(x)$ op die interval, wat 'n globale minimum beteken, en dat $f(d) \geq f(x)$ op die interval, wat 'n globale maksimum tot gevolg het.~~

0

[1]

ii Bepaal f se absolute maksimum (globale maksimum).

Find the absolute maximum (global maximum) of f.

Wenk/Hint: $\frac{d}{dx}\frac{\ln x}{x} = \frac{1-\ln x}{x^2}$.

$$\frac{d}{dx}\frac{\ln x}{x} = \frac{1-\ln x}{x^2}$$

$$f'(x) = 0 \Rightarrow \frac{1-\ln x}{x^2} = 0$$

$$\ln x = 1$$

$$x = e^1$$

2^e^ afgeleide TOETS: $f''(x) = \frac{-\frac{1}{x}\cdot x^2 - (1-\ln x)2x}{x^4}$

$f''(e) = -0{,}049..$ wat < 0

$\therefore$ globale maksimum.

$\therefore$ absolute (globale) maksimum:

$$f(e) = \frac{1}{e}$$

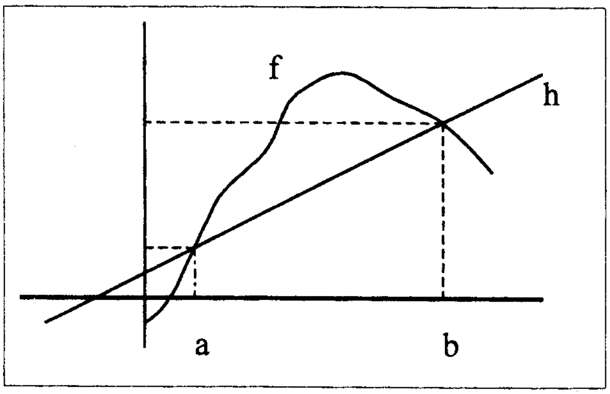
3

[3]

Vraag 16 / Question 16

Laat f 'n kontinue funksie op $[a,b]$ wees en differensieerbaar op (a,b).
Laat h die reguitlyn deur die punte $(a,f(a))$ en $(b,f(b))$ wees.

Let f be a continuous function on $[a,b]$ and differentiable on (a,b).
Let h be the straight line through the points $(a,f(a))$ and $(b,f(b))$.



i Wat is die helling van die reguitlyn h?
What is the gradient of the straight line h?

$$M_h = \frac{f(b)-f(a)}{b-a}$$

1

[1]

ii Pas Rolle se Stelling toe op die funksie $g(x) = f(x) - h(x)$ om die Middelwaarde Stelling te bewys.
Apply Rolle's Theorem to the function $g(x) = f(x) - h(x)$ to prove the Mean Value Theorem.

$g(x) = f(x) - h(x)$
$= f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$

Maar $g(x)$ voldoen ook aan Rolle se Stelling,
want $g(a) = f(a) - \frac{f(b)-f(a)}{b-a}(a-a) - f(a) = 0$
en $g(b) = f(b) - \frac{f(b)-f(a)}{b-a}(b-a) - f(a) = 0$ diff?
$\therefore g(a) = g(b)$, en $g(x)$ is kontinu
Daarom bestaan daar 'n getal c in $(a;b)$ sodat
$g'(c) = 0$ $\therefore f'(c) - \frac{f(b)-f(a)}{b-a} = 0$
$\therefore f'(c) = \frac{f(b)-f(a)}{b-a}$

3

[3]

Vraag 17 / Question 17

Beskou die funksie $f(x) = x\sqrt{x+3}$ met $f'(x) = \dfrac{3x+6}{2\sqrt{x+3}}$ en $f''(x) = \dfrac{3(x+4)}{4(x+3)^{\frac{3}{2}}}$.

Consider the function $f(x) = x\sqrt{x+3}$ with $f'(x) = \dfrac{3x+6}{2\sqrt{x+3}}$ and $f''(x) = \dfrac{3(x+4)}{4(x+3)^{\frac{3}{2}}}$.

i Bepaal, indien moontlik, die interval(le) waarop f daal.
Determine, if possible, the interval(s) on which f is decreasing.

As f daal is $f'(x) < 0$

$\therefore \dfrac{3x+6}{2\sqrt{x+3}} < 0$

$\therefore 3x+6 < 0$

$x < -2$

Maar $f'(x)$ bestaan slegs as $x+3 \geq 0 \therefore x \geq -3$

$\therefore$ f daal op $[-3; -2)$

2

[2]

ii Bepaal, indien moontlik, die interval(le) waarop f konkaaf na bo is.
Determine, if possible, the interval(s) on which f is concave up.

Vir f om konkaaf na bo te wees, moet

$f''(x) > 0$

$\dfrac{3(x+4)}{4(x+3)^{\frac{3}{2}}} > 0$

$3(x+4) > 0$

$x > -4$

Maar vir $f''(x)$ om te bestaan moet

$x+3 \geq 0$

$x \geq -3$

$\therefore$ f is konkaaf na bo op $x \geq -3$

-1

1

[2]

Vraag 18 / Question 18

Bepaal die vergelyking(s) van die horisontale asimptoot(-tote) van $f(x) = \frac{\ln x}{\sqrt{x}}$.

Determine the equation(s) of the horizontal asymptote(s) of $f(x) = \frac{\ln x}{\sqrt{x}}$.

[2]

$f(x) = \frac{\ln x}{\sqrt{x}}$

$\lim_{x \to -\infty} \frac{\ln x}{\sqrt{x}}$ bestaan nie want $f(x)$ bestaan net op $(0;\infty)$

$$\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} = \lim_{x \to \infty} \frac{1}{x \cdot \frac{1}{2}(x)^{\frac{1}{2}}} \quad \left(L'Hospital ; vorm \frac{\infty}{\infty}\right)$$

$$= \lim_{x \to \infty} \frac{1}{\frac{x}{2\sqrt{x}}}$$

$$= \lim_{x \to \infty} \frac{2\sqrt{x}}{x} \quad \left(L'Hospital ; vorm \frac{\infty}{\infty}\right)$$

$$= \lim_{x \to \infty} \frac{2 \cdot \frac{1}{2}(x)^{-\frac{1}{2}}}{1}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{x}}$$

$$= 0$$

$\therefore$ Die vergelyking van die horisontale asimptoot van $f(x) = \frac{\ln x}{\sqrt{x}}$ is $y = 0$

2

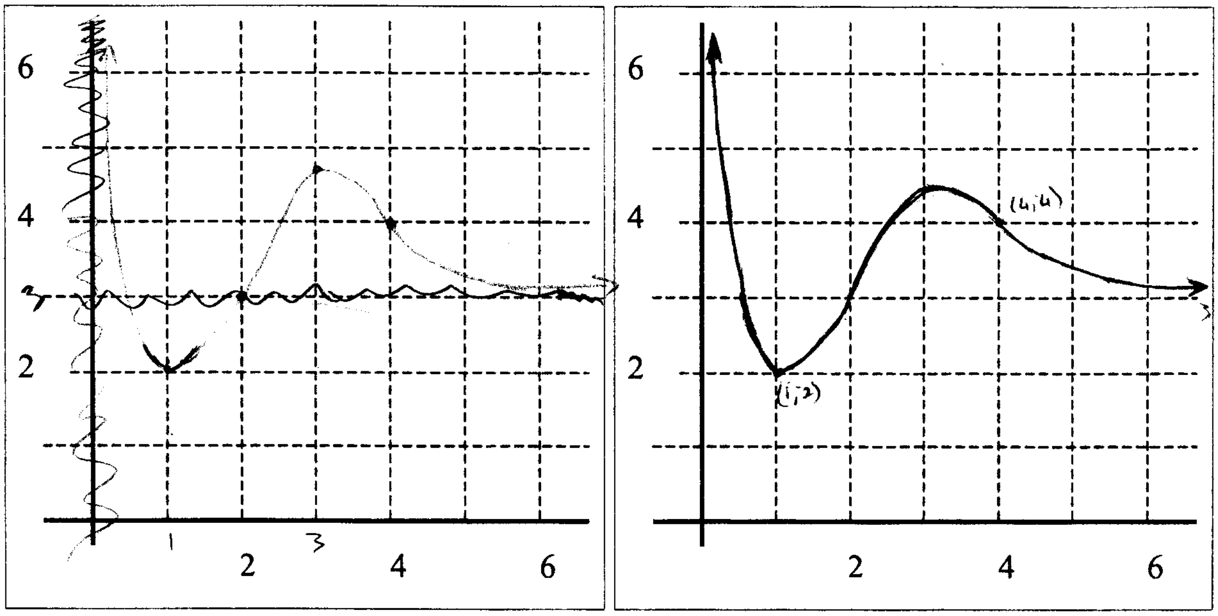
Vraag 19 / Question 19

Skets 'n kontinue funksie $y = f(x)$ vir $x \in (0,\infty)$ met die volgende eienskappe:

- $f(x) > 0$ vir alle $x \in (0,\infty)$.
- $x = 0$ is 'n vertikale asimptoot.
- $f(1) = 2$ is die globale minimum.
- $\lim_{x\to\infty} f(x) = 3$.
- f styg op $(1,3)$ en daal op $(3,\infty)$.
- $f(2) = 3$ en $f(4) = 4$ is infleksiepunte.

Sketch a continuous function $y = f(x)$ for $x \in (0,\infty)$ with the following properties:

- $f(x) > 0$ for all $x \in (0,\infty)$.
- $x = 0$ is a vertical asymptote.
- $f(1) = 2$ is the global minimum.
- $\lim_{x\to\infty} f(x) = 3$.
- f is increasing on $(1,3)$ and decreasing on $(3,\infty)$.
- $f(2) = 3$ and $f(4) = 4$ are points of inflection.



Rofwerk/Scribling

2

[2