UNIVERSITEIT VAN PRETORIA **UNIVERSITY OF PRETORIA**

Departement Wiskunde en Toegepaste Wiskunde **Department of Mathematics and Applied Maths**

4 Mei / May 2002
Punte / Marks: **45**
Tyd / Time: **90 min**

Punte Marks	32

WTW 158: Calculus
SEMESTERTOETS 2 / SEMESTER TEST 2

+2 = 34

Van / Surname:

VOORNAME / FIRST NAMES:

STUDENTNO: HANDTEKENING /SIGNATURE:

STUDIERIGTING/FIELD OF STUDY
BENG CHEMICAL

***Omkring** jou lesinggroepnommer op die tabel / **Encircle** your lecture group number on the table*

Lesinggroepnommer Lecture group number	Dosent Lecturer	Lokaal Venue	Taal Language	Periode Period
1	Mev / Mrs AE du Preez	GW 4-3	Afrikaans	B
2	Mev / Mrs L Mostert	Ing II 3-40	Afrikaans	B
3	Prof MJ Schoeman	Louw	English	B
4	Prof MJ Schoeman	GW 4-3 & GW 4-1	Afrikaans	E
5	Mev / Mrs KH Jordaan	Maths 2-1	English	E

Lees eers die volgende instruksies

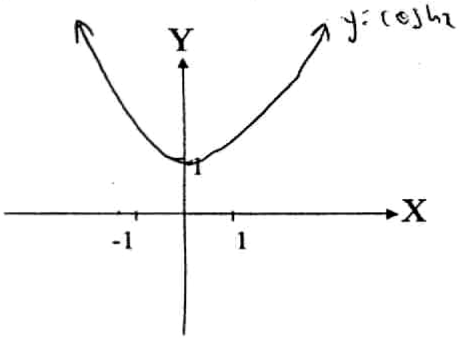
1. *Slegs nie-programmeerbare sakrekenaars mag gebruik word.*
2. *Die vraestel bestaan uit bladsye 1 tot 8(vrae 1-10). Kontroleer of u vraestel volledig is.*
3. *Doen alle krapwerk op die blanko bladsy(e). Dit word nie nagesien nie.*
4. *As u meer as die beskikbare ruimte vir 'n antwoord nodig het, gebruik dan ook die blanko bladsy(e) en dui dit duidelik aan deur 'n **raam daarom te trek.***
5. *Geen potloodwerk word nagesien nie.*
6. *Korreksievloeistof (bv.Tipp-Ex) mag nie gebruik word nie.*
7. *Geen antwoordboek mag uit die lokaal geneem word nie.*
8. *Enige navrae oor die nasienwerk moet binne **drie dae** nadat die toetse terugbesorg is, gedoen word. Daarna word aanvaar dat alles korrek is.*
9. ***Toon alle bewerkings duidelik.***

First read the following instructions

1. *Only non-programmable calculators may be used.*
2. *The question paper consists of pages 1 to 8(questions 1-10). Check if your paper is complete.*
3. *Scribbling can be done on the empty page(s). This will not be marked.*
4. *If you need more than the available space for an answer, you may also use the empty page(s). Indicate it clearly by **framing** it.*
5. *Work in pencil will not be marked.*
6. *Use of correction fluid (e.g.Tipp-Ex) is not allowed.*
7. *No test scripts may be removed from the venues.*
8. *Any queries about the marking must be done within **three days** after the tests have been handed back. After that we assume that everything is in order.*
9. ***Show all computations clearly.***

VRAAG 1 (TEGNIEKE) / *QUESTION 1 (TECHNIQUES)*

(1.1) Skets / *Sketch* $y = \cosh x$.



[1]

(1.2) Sonder om te vereenvoudig, vind $\frac{dy}{dx}$ as

Without simplifying, find $\frac{dy}{dx}$ *if*

(1.2.1) $y = 2^x$

$^{dy}/_{dx} = 2^x \ln 2$

[1]

(1.2.2) $y = \frac{\sin x}{x}$.

$^{dy}/_{dx} = \frac{x\cos x - \sin x}{x^2}$

[1]

(1.2.3) $y = x\sqrt{1-x^2}$

$^{dy}/_{dx} = \sqrt{1-x^2} + x \cdot \tfrac{1}{2}(1-x)^{-1/2} \cdot -2x$

[1]

$10x^{-1}$

(1.2.4) $y = \ln\left(\frac{10}{x}\right)$

$^{dy}/_{dx} = \frac{1}{10/x} \times -10x^{-2}$

$10x^{-1}$

$\frac{-10}{x^2}$

[1]

(1.2.5) $y = \sin^{-1}(2x-3)$ [Notasie / Notation: $\sin^{-1}x = \arcsin x = bg\sin x$]

$^{dy}/_{dx} = \frac{1}{\sqrt{1-x^2(2x-3)}} \times 2$

[1]

(1.2.6) $y = xe^{x^2}$

$dy/dx = e^{x^2} + x.e^{x^2} \cdot 2x$

/ [1]

(1.2.7) $y = \sqrt{\cos(2x)}$

$dy/dx = \cancel{1/2 (\cos 2x)^{-1/2} \cdot 2}$ $dy/dx = 1/2 (\cos 2x)^{-1/2} \cdot -\sin 2x \cdot 2$

/ [1]

(1.2.8) $y = \tan\left(\frac{1}{3x}\right)$

$dy/dx = \sec^2(1/3x) \cdot \frac{-x^{-2}}{3}$

1

/ [1]

(1.2.9) $y = \cosh 7x$

$dy/dx = \sinh 7x \cdot 7$

/ [1]

VRAAG 2 / *QUESTION 2*

Gee die interval waarop $f(x) = \frac{x}{x^2+1}$ kontinu is. Motiveer u antwoord.

Give the interval on which $f(x) = \frac{x}{x^2+1}$ is continuous. Give reasons for your answer.

0 Is ~~continou~~ continuous on $(-\infty, \infty)$, ~~because the domain is~~ because the function is defined for every value of x.

0 [2]

Why?

And??

f is continuous on $\mathbb{R}$ since both x and x^2+1 are polynomials and $x^2+1 \neq 0$ $\forall x$

VRAAG 3 / *QUESTION 3*

Gee die waardes van die volgende limiete.

Give the values of the following limits.

(i) $\lim_{x \to 0} \frac{\tan x}{x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \frac{\sec^2 x}{1} = 1$ ✓

$\frac{x+1}{x}$

(ii) $\lim_{h \to 0} \frac{e^h - 1}{h}$ $\left(\frac{0}{0}\right)$

$= \lim_{h \to 0} \frac{e^h}{1} = 1$ ✓

(iii) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

$= \lim_{x \to \infty} \cancel{\frac{x+1}{x}}$ let $y = (1 + 1/x)^x$

$\ln y = x \ln(1 + 1/x)$

$\lim_{x \to \infty} \ln y = \lim_{x \to \infty} x \ln(1 + 1/x)$

$= \lim_{x \to \infty} \frac{\ln(1 + 1/x)}{x^{-1}}$

$= \lim_{x \to \infty} \frac{\frac{1}{1+1/x} \cdot -x^{-2}}{-x^{-2}}$

$= \lim_{x \to \infty} \frac{\frac{1}{\frac{x+1}{x}} \times -1/x^2}{-1/x^2}$

$= \lim_{x \to \infty} \frac{x}{(x+1)x^2} \times \frac{x^2}{1}$ $\left(\frac{\infty}{\infty}\right)$

~~$0 = \infty$~~

~~$\ln y = \infty$~~

~~$y = e^\infty = \infty$~~

$= \lim_{x \to \infty} \frac{1}{1} = 1$

$\ln y = 1$

$y = e^1$

$\therefore \lim_{x \to \infty} (1 + 1/x)^x = e$ ✓

[3]

3

VRAAG 4 / *QUESTION 4*

Gegee / *Given* $f(x) = \frac{4 - \sqrt{x}}{x - 16}$. $= \frac{-(\sqrt{x} - 4)}{(\sqrt{x} - 4)(\sqrt{x} + 4)}$

Vind die horisontale en vertikale asimptote van f indien moontlik. Toon alle berekeninge.

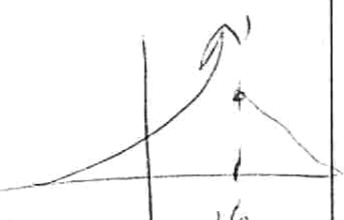
Find the horizontal and vertical asymptotes of f if possible. Show all calculations.

$\lim_{x \to 16^+} \frac{4 - \sqrt{x}}{x - 16}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to 16^+} \frac{-1/2 (x)^{-1/2}}{1}$

$= -1/8$ ✓

$\lim_{x \to 16} f(x) = ?$



3

[5]

$\therefore$ There are no vertical asymptotes

you can't conclude before you've checked both sides

$\lim_{x \to \infty} \frac{4 - \sqrt{x}}{x - 16}$

DX?

$= \lim_{x \to \infty} -1/2 (x)^{-1/2}$ $\frac{-1}{\sqrt{x}}$

$= 0$ ✓

$\therefore y = 0$ is a horizontal asymptote ✓

$$= \frac{-(4-\sqrt{x})}{(4-\sqrt{x})(4+\sqrt{x})} = \frac{-1}{4+\sqrt{x}}, \quad x \neq 16$$

$D_f = [0, \infty) \setminus \{16\}$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{-1}{4+\sqrt{x}} = 0$$

$\therefore y = 0$ is a horizontal asymptote

Possible vertical asymptote at $x = 16$

VRAAG 5 / *QUESTION 5*

Gebruik die definisie van die afgeleide (eerste beginsels) om aan te toon dat $D_x \cos x = -\sin x$.

Use the definition of the derivative (first principles) to show that $D_x \cos x = -\sin x$.

let $y = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cdot \cos h - \cancel{\cos} \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{(\cos h - 1)}{h} - \lim_{h \to 0} \sin x \cdot \cancel{\lim \sin h} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$$

$\therefore D_x \cos x = -\sin x$

[4]

4

VRAAG 6 / *QUESTION 6*

Vind $\frac{dy}{dx}$ in die punt $(-1,1)$ vir die kromme met vergelyking $(x+y)^3 = x^3 + y^3$.

Find $\frac{dy}{dx}$ *in the point* $(-1,1)$ *for the curve with equation* $(x+y)^3 = x^3 + y^3$.

$(x+y)^3 = x^3 + y^3$

Set $y = x^3 + y^3$

Differentiate implicitly : $3(x+y)^2(1 + dy/dx) = 3x^2 + 3y^2 \cdot dy/dx$

$3(x+y)^2 \cdot y' = 3x^2 + 3y^2 y'$

$3(x+y)^2 \cdot y' - 3y^2 y' = 3x^2$

$y'[3(x+y)^2 - 3y^2] = 3x^2$

Subs $(-1,1)$ $3(-1+1)^2(1 + dy/dx) = 3 + 3\,dy/dx$

$y' = \dfrac{3x^2}{3(x+y)^2 - 3y^2}$

$\therefore dy/dx = -1$

$\dfrac{3}{-3} = -1$

4

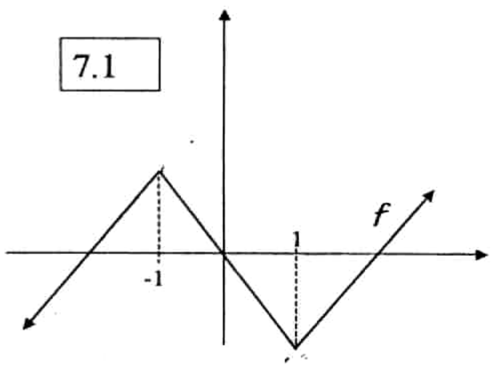
[4]

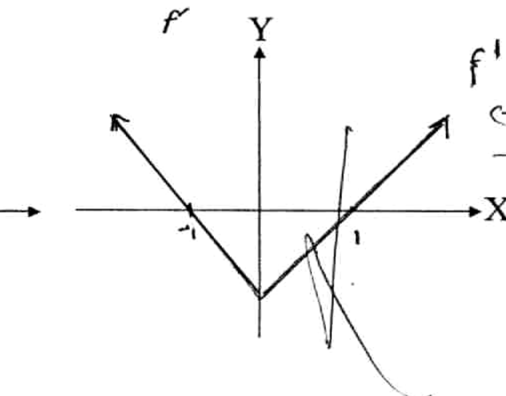
VRAAG 7 / *QUESTION 7*

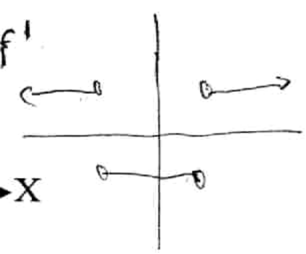
Vir die gegewe funksies f en g, skets hulle afgeleides.

For the given functions f and g, *sketch their derivatives.*

7.1



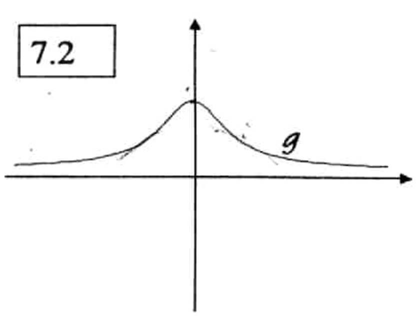


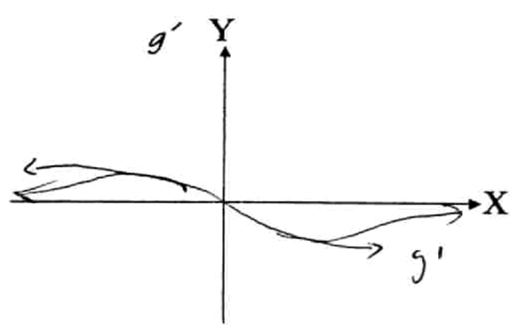


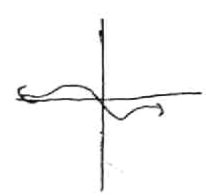
[2]

0

7.2







VRAAG 8 / *QUESTION 8*

Gegee/ *Given*

$g(5) = -3,\ g'(5) = 6,\ h(5) = 3,\ h'(5) = -2$

Gee die waardes van $f'(5)$ as

Give the values of $f'(5)$ if

(8.1) $f(x) = (gh)(x)$

product

$(g \circ h)(x)$

$f'(x) = g'(h(x)).h'(x)$

$f'(5) = g'(h(5)).h'(5)$

$f'(5) = g'(3).(-2)$

$f'(5) = 24$

(8.2) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2.g'(x)$

$f'(5) = 3(g(5))^2 \cdot g'(5)$

$= 3.(-3)^2.6$

$= 162$

$f'(x) = 0$

$2\cos x + 2\sin 2x = 0$

$\cos x + 2\sin x\cos x = 0$

$\cos x = 0$ or $\sin x = -1/2$

$x = \pi/2$ no sol.

$f(0) = -1$

$f(\pi) = -1$ $\therefore$ Absolute min is -1 at $x = 0$ $x = \pi$

[2]

VRAAG 9 / *QUESTION 9*

Laat / *Let* $f(x) = 2\sin x - \cos 2x,\quad x \in [0, \pi]$.

mixing degrees and radians.

(9.1) Verduidelik waarom f absolute ekstreemwaardes besit.

Explain why f has absolute extreme values.

because it has a closed interval

f is continuous on a closed interval

[1]

(9.2) Vind die absolute minimum van f.
Find the absolute minimum of f.

$f'(x) = 2\cos x + \sin 2x \times 2$

$f'(x) = 0 \quad 2\cos x + 2\sin 2x = 0$

~~$\sin 2x =$~~

$2\cos x + 2\sin x\cos x = 0$

$\cos x(1 + 2\sin x) = 0$

$\cos x = 0$ or $\sin x = -1/2$

$x = \pm\pi/2 + 360n,\ n \in \mathbb{Z}$ (2π)

$x \in \{\pi/2\}$ NO!

$\sin x = -1/2$

$x = -\pi/6 + 360n,\ n \in \mathbb{Z}$

$x = \frac{7\pi}{6} + 360n,\ n \in \mathbb{Z}$

No Sol.

$f(\pi/2) = \cancel{2\sin(2 \cdot \pi/2)}\ 2\sin\pi/2 - \cos(2\cdot\pi/2)$

$= 3$

$f(0) = -1$

$f(\pi) = 2\sin\pi - \cos(2\pi) = -1$

$\therefore$ Absolute minimum is $(0;-1) \cup (\pi,-1$

[4]

VRAAG 10 / *QUESTION* 10

Vir / *For* $f(x) = (x^2 - 4)^{2/3}$ is $f'(x) = \frac{4x}{3(x^2-4)^{1/3}}$.

(i) Gee die kritieke getalle van f.

Give the critical numbers of f.

$f'(x) \nexists$ at $x = \pm 2$

$f'(x) = 0$ at $x = 0$

$\therefore$ Critical numbers are $x = 0$ or $x = \pm 2$

2 [2]

(ii) Vind die relatiewe ekstreemwaardes van f. Toon alle berekeninge.

Find the relative extreme values of f. Show all computations.

~~f(0) = 0~~ $f(0) = 2{,}5$ $\therefore$ Local max $(0, 2.5)$

x	$x < -2$	-2	$-2 < x < 0$	0	$0 < x < 2$	2	$x > 2$
$f'(x)$	$-$	und	$+$	0	$-$	und.	$+$
$f(x)$	$\searrow$		$\nearrow$	$-$	$\searrow$		$\nearrow$

Min?

[3]

2

$f(x)$ increases on ~~(0,2) (2,∞)~~ $(-2, 0)$ and $(2, \infty)$

$f(x)$ decreases on ~~(-∞,-2) (-2,0)~~ $(-\infty, -2)$ and $(0, 2)$

x-int $y = 0$

$x = \pm 2$

$(2,0)\ (-2,0)$

y-int: $x = 0$

$y = 2{,}5$

$(0, 2.5)$

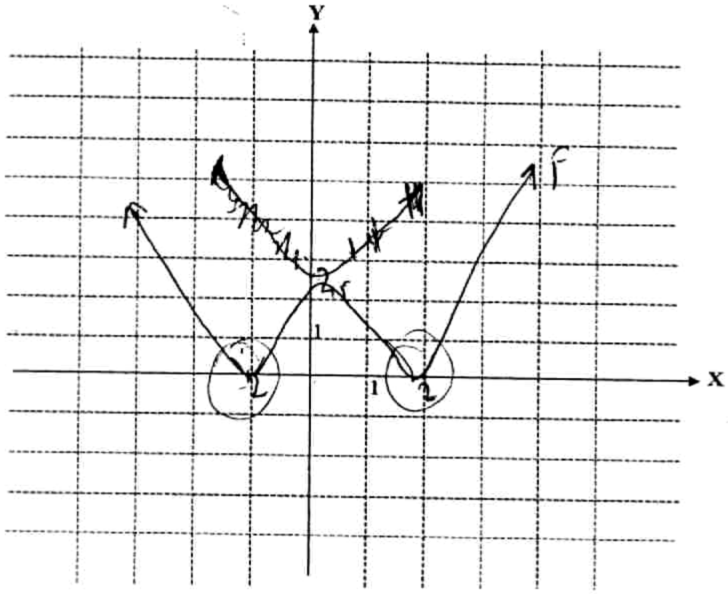
local max $(x = 0)$ or $y = 2.52$

local min at $x = -2$ & $x = 2$ or $y = 0$

At $x = \pm 2$ derivative ~~the~~ is not defined so we have a sharp point (cusp)

(iii) Skets f sonder om asimptote, buigpunte en konkawiteit in ag te neem.

Sketch f without considering asymptotes, inflection points and concavity.



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[3]