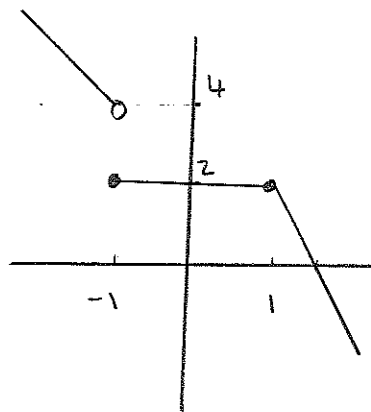


# Memo short questions, WTW158, sem test 1 2008.

## Question 1

$$f(x) = \begin{cases} 2 & \text{if } -1 \leq x \leq 1 \\ 3-x & \text{if } x > 1 \text{ or } x < -1 \end{cases}$$

$$= \begin{cases} 3-x & \text{if } x < -1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ 3-x & \text{if } x > 1 \end{cases}$$



Domain =  $\mathbb{R}$       Range =  $(-\infty, 2] \cup (4, \infty) = \mathbb{R} \setminus (2, 4]$

## Question 2

$$\begin{aligned} h(-x) &= \frac{f(-x)}{g(-x)} = \frac{(-x)^3 - 4(-x)}{1 + \cos(-x)} = \frac{-x^3 + 4x}{1 + \cos x} = -\frac{[x^3 - 4x]}{1 + \cos x} \\ &= -\frac{f(x)}{g(x)} = -h(x) \end{aligned}$$

## Question 3

$$y = e^x$$

$$y = e^{-x}$$

$$y = -e^{-x}$$

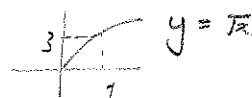
$$y = 10 - e^{-x}$$

## Question 4

$$x-1 \geq 0 \text{ and } 3-\sqrt{x-1} > 0 \Rightarrow x \geq 1 \text{ and } \sqrt{x-1} < 3$$

$$\Rightarrow x \geq 1 \text{ and } 0 \leq x-1 < 9$$

$$\Rightarrow x \geq 1 \text{ and } 1 \leq x < 10 \Rightarrow x \in [1, 10)$$



## Question 5

$$\begin{aligned} \text{let } y &= \arcsin\left(-\frac{1}{\sqrt{2}}\right) \Rightarrow \sin y = -\frac{1}{\sqrt{2}} \text{ and } y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ &\Rightarrow y = -\frac{\pi}{4} \end{aligned}$$

$$\cos(\arcsin(-\frac{1}{\sqrt{2}})) = \cos y = \cos(-\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$$

### Question 6

$$f: [-1, 1] \rightarrow [0, 2] \Rightarrow f(x-1): [0, 2] \rightarrow [0, 2]$$

$$\Rightarrow g(x) = f(x-1) + 1: [0, 2] \rightarrow [1, 3]$$

$$\Rightarrow g^{-1}: [1, 3] \rightarrow [0, 2]$$

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### Question 7

$$\lim_{x \rightarrow \infty} \frac{1 + e^{-11x}}{1 + e^{-x}} = \frac{1 + 1}{1 + 0} = 2$$

$$\lim_{x \rightarrow \infty} \frac{1 + e^{-\frac{1}{x}}}{1 + e^{-x}} = 0$$

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### Question 8

$$\begin{aligned} \lim_{x \rightarrow 1^-} \frac{x-1}{1-x-1} &= \lim_{x \rightarrow 1^-} \frac{x-1}{-(x-1)} & x < 1 \Rightarrow x-1 < 0 \\ &= \lim_{x \rightarrow 1^-} -1 = -1 \end{aligned}$$

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### Question 9

$f(a)$  is defined.

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### Question 10

$$f(0) = a$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$