

Beantwoord vrae 1 tot 16 op die MERKLESERVORM se KANT 1 /

Answer questions 1 to 16 on the OPTIC READER FORM on SIDE 1

Vraag 1 / Question 1

As / If $f(x) = \sin^3 5x$ dan / then $f'(x) =$

| | |
|------------------------------------|-----------------------------------|
| (1 a) $3 \sin^2 5x$ | (1 b) $15 \cos^2 5x$ |
| (1 c) $3 \sin^2 5x (-\cos 5x) (5)$ | (1 d) $3 \sin^2 5x (\cos 5x) (5)$ |

Ans: $f'(x) = 3 (\sin^2 5x) (\cos 5x) (5)$

(1 d)

Vraag 2 / Question 2

As / If $f(x) = 2 \sin x \cos x$ dan / then $f'(x) =$

| | | | |
|--------------------|-------------------|-----------------|--------------------|
| (2 a) $2 \cos^2 x$ | (2 b) $2 \cos 2x$ | (2 c) $\sin 2x$ | (2 d) $-2 \cos 2x$ |
|--------------------|-------------------|-----------------|--------------------|

Ans: $f(x) = 2 \sin x \cos x = \sin 2x \therefore f'(x) = (\cos 2x) (2)$

of / or $f'(x) = 2(\cos x \cos x + \sin x (-\sin x)) = 2(\cos^2 x - \sin^2 x) = 2 \cos 2x$

(2 b)

Vraag 3 / Question 3

As / If $f(x) = e^{\cosh x}$ dan / then $f'(x) =$

| | | | |
|----------------------|--------------------------------|-------------------------------|---------------------|
| (3 a) $e^{-\sinh x}$ | (3 b) $e^{\cosh x} (-\sinh x)$ | (3 c) $e^{\cosh x} (\sinh x)$ | (3 d) $e^{\sinh x}$ |
|----------------------|--------------------------------|-------------------------------|---------------------|

Ans: $f'(x) = e^{\cosh x} (\sinh x)$

(3 c)

Vraag 4 / Question 4

As / If $f(t) = 2^t$ dan / then $f'(t) =$

| | | | |
|---------------------|---------------------------|-------------------|-------------|
| (4 a) $(\ln 2) 2^t$ | (4 b) $\frac{2^t}{\ln 2}$ | (4 c) $t 2^{t-1}$ | (4 d) 2^t |
|---------------------|---------------------------|-------------------|-------------|

Ans: $\ln f(t) = \ln 2^t = t \ln 2 \therefore \frac{f'(t)}{f(t)} = \ln 2 \Leftrightarrow f'(t) = (\ln 2) 2^t$

(4 a)

Vraag 5 / Question 5

$D_t \left(\frac{\sin t}{t} \right) =$

| | | | |
|---------|---------------------------------------|---------|---------------------------------------|
| (5 a) 1 | (5 b) $\frac{t \cos t - \sin t}{t^2}$ | (5 c) 0 | (5 d) $\frac{\sin t - t \cos t}{t^2}$ |
|---------|---------------------------------------|---------|---------------------------------------|

Ans: $D_t \left(\frac{\sin t}{t} \right) = \frac{(\cos t)(t) - \sin t (1)}{t^2}$

(5 b)

Vraag 6 / Question 6

Bepaal / Find: $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$

| | |
|--|--|
| (6 a) die limiet bestaan nie / the limit does not exist | (6 b) $\lim_{x \rightarrow 0} \frac{3 \sin x}{5 \sin x} = \frac{3}{5}$ |
| (6 c) $\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \times \frac{5x}{\sin 5x} \times \frac{3}{5} = \frac{3}{5}$ | (6 d) 0 |

Ans: $\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \times \frac{5x}{\sin 5x} \times \frac{3}{5} = \lim_{3x \rightarrow 0} \frac{\sin 3x}{3x} \times \lim_{5x \rightarrow 0} \frac{5x}{\sin 5x} \times \frac{3}{5} = 1 \times 1 \times \frac{3}{5} = \frac{3}{5}$

(6 c)

Vraag 7 / Question 7

$D_x^{31} (\cos 2x) =$

| | | | |
|---------------------------|------------------------|---------------------------|-----------------|
| (7 a) $(-2)^{31} \sin 2x$ | (7 b) $2^{31} \sin 2x$ | (7 c) $(-2)^{31} \cos 2x$ | (7 d) $\sin 2x$ |
|---------------------------|------------------------|---------------------------|-----------------|

Ans: $D_x(\cos 2x) = (-\sin 2x)(2) = -2\sin 2x,$

$D_x^2(\cos 2x) = \frac{d}{dx}(-2\sin 2x) = -2(\cos 2x) \times 2 = -4\cos 2x$

$D_x^3(\cos 2x) = \frac{d}{dx}[-4\cos 2x] = -4(-\sin 2x) \times 2 = 8\sin 2x$

$D_x^4(\cos 2x) = \frac{d}{dx}[8\sin 2x] = 8(\cos 2x) \times 2 = 16\cos 2x$

\vdots

$D_x^{31}(\cos 2x) = (-1)^{16} 2^{31} \sin 2x = 2^{31} \sin 2x$

$\left[\text{want / since } D_x^{2n}(\cos 2x) = (-1)^n 2^{2n} \cos 2x \text{ en / and } D_x^{2n+1}(\cos 2x) = (-1)^{n+1} 2^{2n+1} \sin 2x \right]$

(7 b)

Vrae 8 & 9 / Questions 8 & 9

Vir funksies f en g word gegee dat / For functions f and g it is given that

$f(2) = 4, \quad f'(2) = 3, \quad f'(5) = 3,$

$g(2) = 5, \quad g'(2) = 1, \quad g'(4) = 2.$

Dan / Then $(fg)'(2) =$

| | | | |
|---------|----------|---------|---|
| (8 a) 3 | (8 b) 19 | (8 c) 2 | (8 d) kan nie bereken word nie / can not be found |
|---------|----------|---------|---|

Ans: $(fg)'(2) = (f'(2))g(2) + (f(2))g'(2) = 3 \times 5 + 4 \times 1 = 19$

(8 b)

en / and $(f \circ g)'(2) =$

| | | | |
|---------|----------|---------|---|
| (9 a) 3 | (9 b) 19 | (9 c) 2 | (9 d) kan nie bereken word nie / can not be found |
|---------|----------|---------|---|

Ans: $(f \circ g)'(2) = f'(g(2))g'(2) = f'(5)g'(2) = 3 \times 1$

(9 a)

Vraag 10 / Question 10

Watter funksie se afgeleide is $\sec^2 3x$? / Which function's derivative is $\sec^2 3x$?

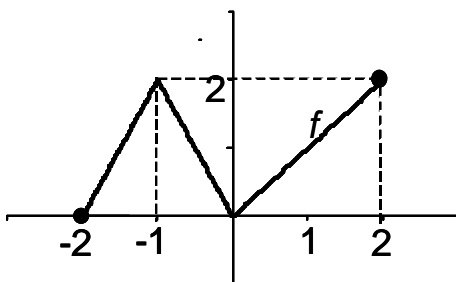
| | | | |
|---------------------------|-------------------|----------------------------|----------------------------|
| (10 a) $6\sec 3x \tan 3x$ | (10 b) $2\sec 3x$ | (10 c) $\frac{\tan 3x}{3}$ | (10 d) $\frac{\tan 3x}{6}$ |
|---------------------------|-------------------|----------------------------|----------------------------|

Ans: $\frac{d}{dx}\left(\frac{\tan 3x}{3}\right) = \frac{1}{3} \frac{d}{dx}(\tan 3x) = \frac{1}{3}(\sec^2 3x)(3) = \sec^2 3x$

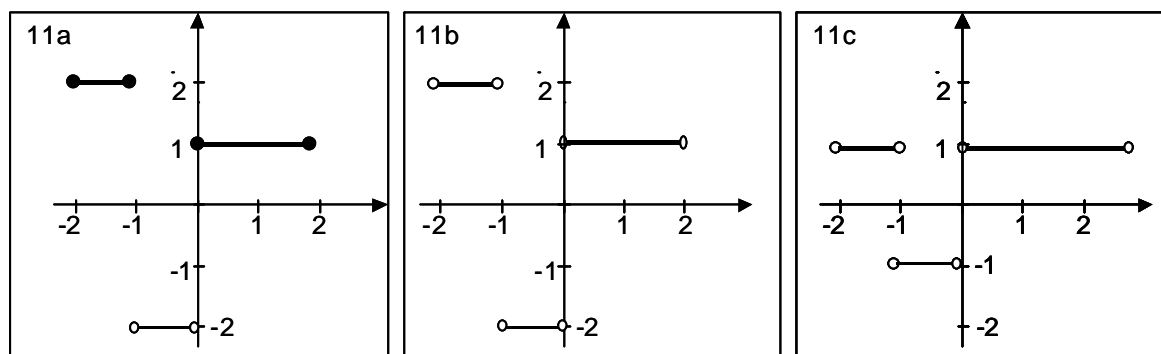
(10 c)

Vraag 11 / Question 11

Die grafiek van 'n funksie f word gegee. / The graph of a function f is given.



'n Moontlike grafiek vir f' is / A possible graph of f' is



(11 d) nie een van hierdie nie / none of these

Ans: f is nie differensieerbaar in $x = -1$ en $x = 0$ en die helling van f is onderskeidelik 2 op $(-2, -1)$, -2 op $(-1, 0)$ en 1 op $(0, 2)$.

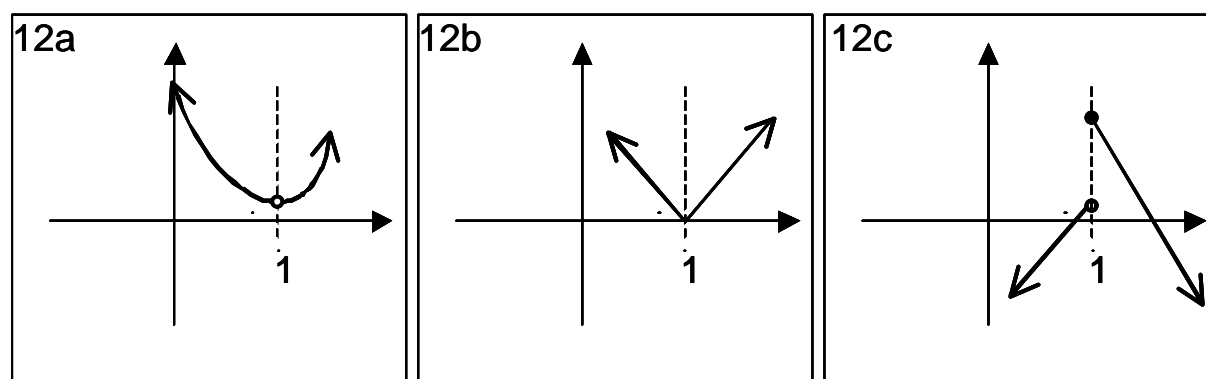
f is not differentiable at $x = -1$ and $x = 0$ and the tangent of f is respectively 2 on $(-2, -1)$, -2 on $(-1, 0)$ and 1 on $(0, 2)$.

(11b)

Vraag 12 / Question 12

'n Moontlike grafiek van 'n funksie wat differensieerbaar in $x = 1$ is, maar nie kontinu in $x = 1$ is nie, word gegee deur

A possible graph of a function that is differentiable at $x = 1$ but not continuous at $x = 1$ is given by



(12 d) daar is nie so 'n funksie nie / there is no such function

Ans: As 'n funksie differensieerbaar is, is die funksie ook kontinu. (Stelling 4 p 171)
Daar is dus nie so 'n funksie nie.

If a function is differentiable, then it is also continuous (Theorem 4 p 171)

Therefore there is no such a function

(12d)

Vrae 13 & 14 / Questions 13 & 14

Bepaal $\frac{dy}{dx}$ as $\sin(x+y) = y$. / Find $\frac{dy}{dx}$ if $\sin(x+y) = y$.

| | | |
|--|----------|--------------------|
| (13 a) $\frac{-\cos(x+y)}{\cos(x+y)-1}$ | (13 b) 1 | (13 c) $\cos(x+y)$ |
| (13 d) nie een van hierdie nie / none of these | | |

$$\begin{aligned} \text{Ans: } \frac{d}{dx}(\sin(x+y)) &= \frac{d}{dx}(y) \Leftrightarrow [\cos(x+y)](1 + \frac{dy}{dx}) = \frac{dy}{dx} \\ \Leftrightarrow [\cos(x+y)] \frac{dy}{dx} - \frac{dy}{dx} &= -\cos(x+y) \Leftrightarrow ([\cos(x+y)] - 1) \frac{dy}{dx} = -\cos(x+y) \\ \Leftrightarrow \frac{dy}{dx} &= \frac{-\cos(x+y)}{\cos(x+y)-1} \end{aligned}$$

(13 a)

Die vergelyking van die raaklyn in die punt $(0,0)$ aan die grafiek van $\sin(x+y) = y$ is
The equation of the tangent line at $(0,0)$ to the graph of $\sin(x+y) = y$ is

| | | |
|--|----------------|----------------|
| (14 a) $y = 0$ | (14 b) $x = 0$ | (14 c) $y = x$ |
| (14 d) nie een van hierdie nie / none of these | | |

Ans: $\frac{dy}{dx} \Big|_{(x,y)=(0,0)} = \frac{-\cos(0+0)}{\cos(0+0)-1} = \frac{-1}{1-1}$. (die helling van die raaklyn neig na oneindig / the gradient of the tangent line tends to infinity)
 \therefore 'n Vertikale raaklyn, nl. $x = 0$ / A vertical tangent line i.e. $x = 0$.

(14 b)

Vrae 15 & 16 / Questions 15 & 16

Laat / Let $f(x) = e^{-x} - e^{-2x}$, $x \in [0, 1]$.

dan / then $f'(x) = 0$ as / if $x =$

| | | |
|--|----------------|--------------------------|
| (15 a) 0 of / or 1 | (15 b) $\ln 2$ | (15 c) $\ln \frac{1}{2}$ |
| (15 d) nie een van hierdie nie / none of these | | |

Ans: $f'(x) = \frac{d}{dx}(e^{-x} - e^{-2x}) = -e^{-x} - (-2)e^{-2x} = -e^{-2x}(e^x - 2)$
 $\therefore f'(x) = 0 \Leftrightarrow -e^{-2x}(e^x - 2) \Leftrightarrow -e^{-2x} = 0$ (n.v.t. / n.a.) of / or $e^x - 2 = 0$
en /and $e^x - 2 = 0 \Leftrightarrow e^x = 2 \Leftrightarrow x = \ln 2$

(15 b)

Die globale(absolute) minimum van f word bereik in $x =$

The absolute minimum of f is reached at $x =$

| | | | |
|----------|----------|----------------|--|
| (16 a) 1 | (16 b) 0 | (16 c) $\ln 2$ | (16 d) nie een van hierdie nie / none of these |
|----------|----------|----------------|--|

Ans: 2de afgeleide toets: / 2nd derivative test:

$$f''(x) = \frac{d}{dx}[-e^{-2x}(e^x - 2)] = \frac{d}{dx}[-e^{-x} + 2e^{-2x}] = e^{-x} - 4e^{-2x} = e^{-x}(1 - 4e^{-x})$$

$$\therefore f''(\ln 2) = e^{-\ln 2}(1 - 4e^{-\ln 2}) = e^{\ln(\frac{1}{2})}(1 - 4e^{\ln(\frac{1}{2})}) = \frac{1}{2}(1 - 4 \times \frac{1}{2}) = -\frac{1}{2} < 0$$

$\therefore f(\ln 2)$ is 'n lokale maksimum / $f(\ln 2)$ is a local maximum

[of 1ste afgeleide toets: / 1st derivative test: $f'(x) > 0 \Leftrightarrow -e^{-2x}(e^x - 2) > 0 \Leftrightarrow e^x - 2 < 0$

$$\Leftrightarrow x < \ln 2 \approx 0.6931472$$

$\therefore f$ styg op $(0, \ln 2)$ en f daal op $(\ln 2, 1)$

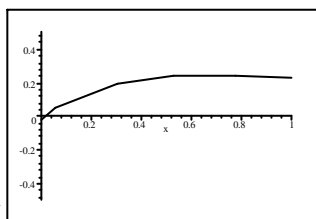
$\therefore f$ increases on $(0, \ln 2)$ and f decreases on $(\ln 2, 1)$

$\therefore f(\ln 2)$ is 'n lokale maksimum / $f(\ln 2)$ is a local maximum]

$$\text{maar / but } f(0) = e^{-0} - e^{-2(0)} = 0 \text{ en / and } f(1) = e^{-1} - e^{-2(1)} = \frac{1}{e} - \frac{1}{e^2} \approx 0.233$$

\therefore globale(absolute) minimum word in $x = 0$ bereik.

\therefore absolute minimum is reached in $x = 0$.



Die grafiek van $f(x) = e^{-x} - e^{-2x}$ / the graph of $f(x) = e^{-x} - e^{-2x}$

(16 b)

Totaal / Total : [16]

BEANTWOORD ALLE VERDERE VRAE OP HIERDIE VRAESTEL en TOON alle bewerkings duidelik aan
ANSWER ALL THE FOLLOWING QUESTIONS ON THE SCRIPT and SHOW all computations clearly.

Vraag 17 / Question 17

Laat / Let $f(x) = x - 2\sin x$, $x \in (0, 3\pi)$.

Gegee dat / Given that $f'(x) = 1 - 2\cos x$ en / and $f''(x) = 2\sin x$.

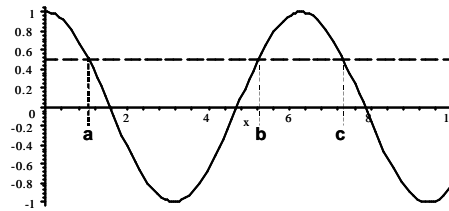
(17.1) Vind die interval(le), indien enige, waarop f daal.

Find the interval(s), if any, on which f is decreasing.

Ans: $f'(x) = 1 - 2\cos x < 0$

$\cos x > \frac{1}{2}$, $\cos x = \frac{1}{2}$ as / if $x = \frac{\pi}{3}$

dan / then $a = \frac{\pi}{3}$, $b = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ en / and $c = 2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$



Hulpgrafiek van $y = \cos x$ / the graph of $y = \cos x$

$\therefore f$ daal op / f decreases on $(0, \frac{\pi}{3})$ en op / and on $(\frac{5\pi}{3}, \frac{7\pi}{3})$

[4]

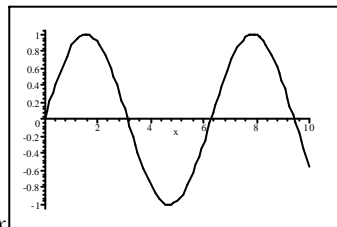
(17.2) Vind die interval(le), indien enige, waarop f konkaaf na bo is.

Find the interval(s), if any, on which f is concave upward.

Ans: f is konkaaf na bo as / f is concave upward if

$f''(x) = 2\sin x > 0 \Leftrightarrow \sin x > 0$

$\therefore f$ is konkaaf na bo op / f is concave upward on $(0, \pi)$ en op / and on $(2\pi, 3\pi)$



Hulpgrafiek van $y = \sin x$ / the graph of $y = \sin x$

[2]

(17.3) Gee die x -waarde(s), indien enige, waar die grafiek van f buigpunte het.

Give the x -value(s), if any, where the graph of f has inflection points.

Ans: f het MOONTLIKE buigpunte waar $f''(x) = 0$ of waar $f''(x) \nexists$.

f has POSSIBLE inflection points where $f''(x) = 0$ or where $f''(x) \nexists$.

f het buigpunte waar f se konkawiteit verander.

f has inflection points where the concavity changes.

$f''(x) = 0 \Leftrightarrow \sin x = 0$: x -waarde(s) / x -value(s): $x = \pi, 2\pi$ in $(0, 3\pi)$.

Vraag 18 / Question 18

Skets 'n funksie f wat kontinu is op \mathbb{R} en die volgende voorwaardes bevredig.

Sketch a function f that is continuous on \mathbb{R} and satisfies the following conditions.

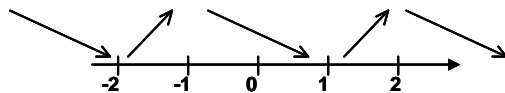
- ♦ $f'(-1) = f'(1) = 0$
- ♦ $f'(x) > 0$ as / if $x \in (-2, -1) \cup (1, 2)$
- ♦ $f''(x) < 0$ as / if $-2 < x < 0$
- ♦ $f'(x) < 0$ as / if $|x| < 1$
- ♦ $f'(x) = -1$ as / if $|x| > 2$
- ♦ Buigpunt / Inflection point $(0, 1)$

Ans:

| | |
|--|---|
| ♦ $f'(-1) = f'(1) = 0$ | moontlike ekstreme / possible extremes |
| ♦ $f'(x) > 0$ as / if $x \in (-2, -1) \cup (1, 2)$ | f styg op / increases on $(-2, -1)$ en op / and on $(1, 2)$ |
| ♦ $f'(x) < 0$ as / if $ x < 1$ | f daal op / decreases on $(-1, 1)$ |
| ♦ $f'(x) = -1$ as / if $ x > 2$ | |
| $f(x) = -x + c$ ('n reguit lyn / a straight line) op / on $(-\infty, -2)$ en op / and on $(2, \infty)$ | |
| ♦ $f''(x) < 0$ as / if $-2 < x < 0$ | f is konkaf na onder op / concave downward on $(-2, 0)$ |
| ♦ Buigpunt / Inflection point $(0, 1)$ | |

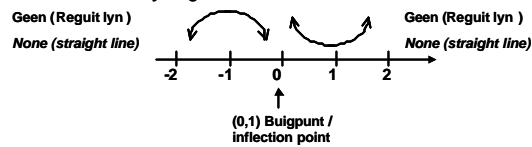
Styg en daal diagram vir f

increasing and decreasing diagram for f



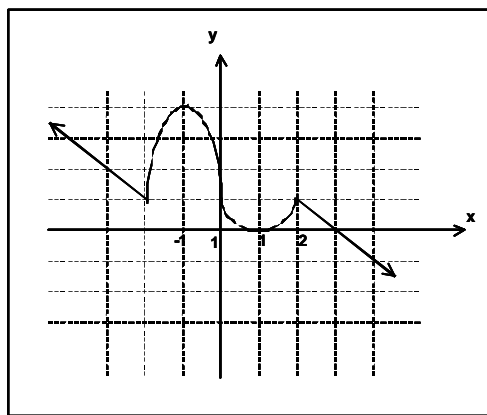
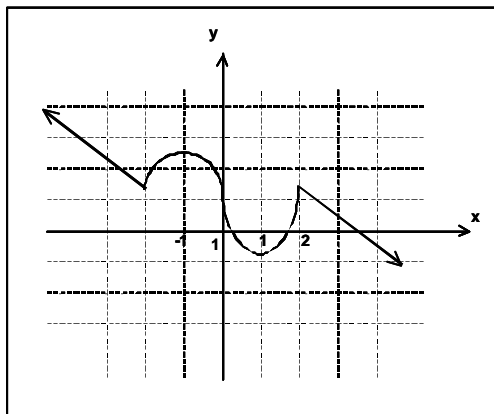
Konkawiteits diagram vir f

concavity diagram for f



Baie

moontlike antwoorde / Many possible answers



Vraag 19 / Question 19

Bepaal / Find $\frac{dy}{dx}$ as / if $y = x^{\cos x}$, $x > 0$.

Ans: $\ln(y) = \ln(x^{\cos x}) = \cos x \ln x$

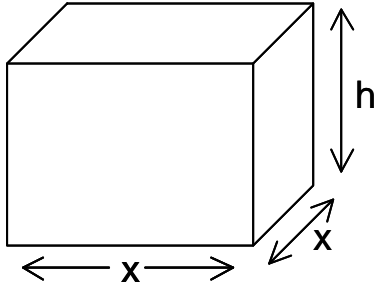
$$\begin{aligned} \frac{d}{dx} [\ln y] &= \frac{d}{dx} [\cos x \ln x] \Leftrightarrow \frac{1}{y} \frac{dy}{dx} = -\sin x \ln x + \cos x \frac{1}{x} \\ \Leftrightarrow \frac{dy}{dx} &= y [-\sin x \ln x + \cos x \frac{1}{x}] = x^{\cos x} [-\sin x \ln x + \frac{\cos x}{x}] \end{aligned}$$

[3]

Vraag 20 / Question 20

'n Houer met 'n vierkantige basis en oop bo, het 'n volume van 32000 cm^3 . Bepaal die afmetings van die houer sodat die minimum hoeveelheid materiaal gebruik word.

A box with a square base and an open top, has a volume of 32000 cm^3 . Find the dimensions of the box such that a minimum amount of material is used.



Ans:

Gegee / Given: $Vol = 32000 \quad \therefore x^2 h = 32000$

Verband / Relation: $\therefore h = \frac{32000}{x^2}$

Gevra / Asked: Buiteoppervlakte / Outer surface

$$S(x) = x^2 + 4xh = x^2 + 4x \left(\frac{32000}{x^2} \right) = \frac{x^3 + 128000}{x}$$

$$\therefore S'(x) = \frac{d}{dx} \left(\frac{x^3 + 128000}{x} \right) = 2 \frac{x^3 - 64000}{x^2}$$

$$S'(x) = 0 \Leftrightarrow \frac{x^3 - 64000}{x^2} = 0 \Leftrightarrow$$

$$x^3 - 64000 = 0 \Leftrightarrow x = 40, \quad (x = 0 \text{ n.v.t. / n.a.})$$

$$\text{Toets / Test } S''(x) = \frac{d}{dx} \left(2 \frac{x^3 - 64000}{x^2} \right) = 2 \frac{x^3 + 128000}{x^3}$$

$$\therefore S''(40) = 2 \frac{40^3 + 128000}{40^3} = 6 > 0 \therefore \text{minimum}$$

$$\therefore \text{Afmetings / Dimensions: } h = \frac{32000}{40^2} = 20 \text{ cm}$$

Afmetings van houer moet / Dimensions of box should be $40 \text{ cm} \times 40 \text{ cm} \times 20 \text{ cm}$.

[4]