WTW158 - Memo, Semestertest 1, 2007

Beantwoord vrae 1 tot 13 op die MERKLEESVORM se KANT 2.
Answer questions 1 to 13 on the OPTIC READER FORM on SIDE 2 (calculations on pp9,10)

Vraag 1 / Question 1

Die waardeversameling van die funksie $y = \text{bgtan}(x) = \tan^{-1}x$ is

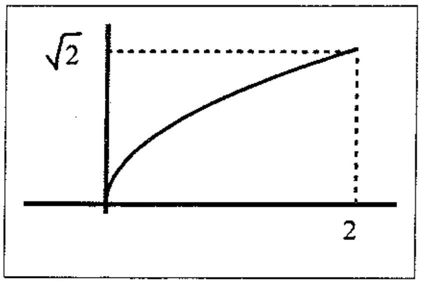
The range of the function $y = \arctan(x) = \tan^{-1}x$ is

[1a] R	[1b] $[-1,1]$	[1c] $[-\frac{\pi}{2},\frac{\pi}{2}]$	[1d] $(-\frac{\pi}{2},\frac{\pi}{2})$	[1e] Geen van hierdie/None of these

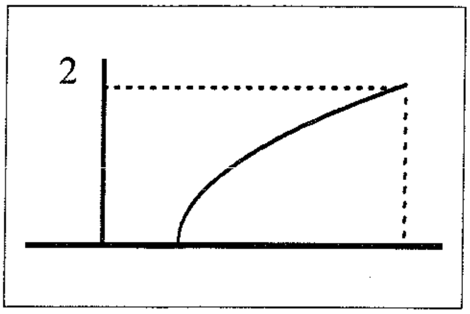
Vraag 2 / Question 2

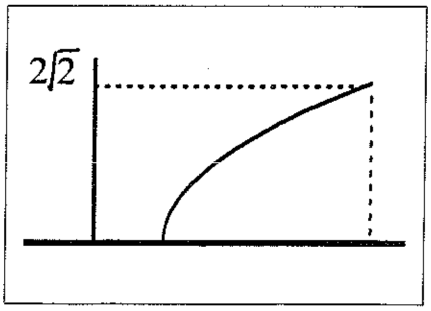
Die grafiek van $f(x) = \sqrt{x},\ x \in [0,2]$ word in die skets hieronder gegee.

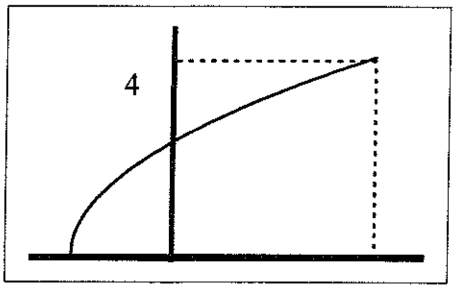
The graph of $f(x) = \sqrt{x},\ x \in [0,2]$ is shown in the sketch below.

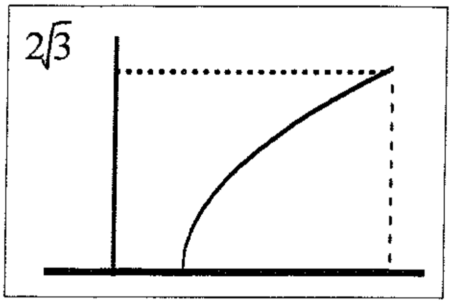


Die grafiek van $2f(x-1)$ is / The graph of $2f(x-1)$ is

[2a])


[2b]


[2c]


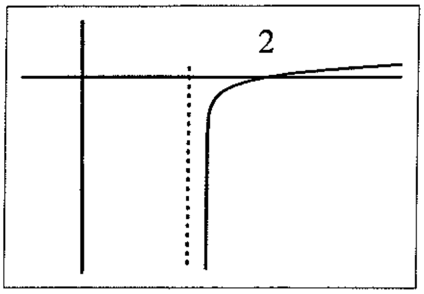
[2d]


Vraag 3 / Question 3

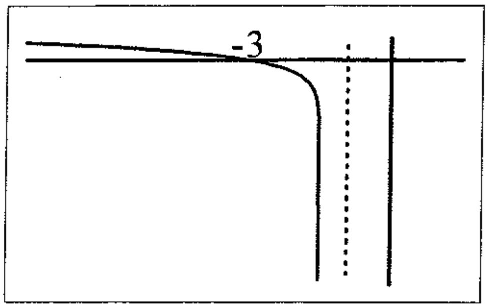
Die grafiek van $f(x) = \ln(x-2)$ word om die lyn $x = 3$ gereflekteer.
Die resulterende grafiek is

The graph of $f(x) = \ln(x-2)$ is reflected about the line $x = 3$.
The resulting graph is

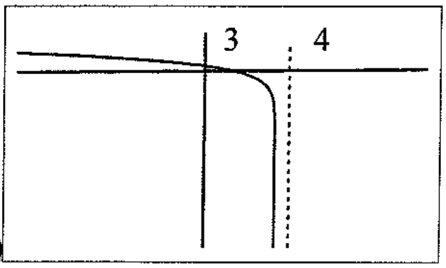
[3a]



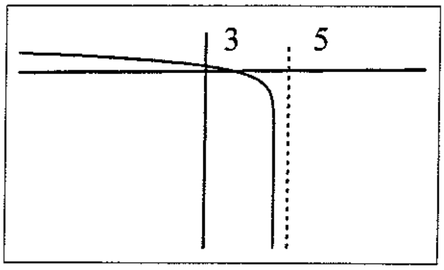
[3b]



[3c]



[3d]



[3e] Geen van hierdie/None of these

Vraag 4 / Question 4

Indien $F(x) = (\sqrt{x^3} + 6)^{10}$ en $f(x) = x^3 + 6$ en $F = g \circ f$ dan is $g(x) =$

If $F(x) = (\sqrt{x^3} + 6)^{10}$ and $f(x) = x^3 + 6$ and $F = g \circ f$ then $g(x) =$

[4a] $(\sqrt{x} + 6)^{10}$	[4b] $(\sqrt{x+6})^{10}$	[4c] x^{10}	[4d] $x^{10} + 6$	[4e] Geen van hierdie/None of these

Vraag 5 / Question 5

Indien / If $a \neq 0$ dan / then $\lim_{x \to 0} \frac{ax^2}{\sin x}$

[5a] $= \frac{1}{a}$	[5b] $= a$	[5c] $= 0$	[5d] is ongedefinieerd / is undefined
[5e] bestaan nie / does not exist		[5f] Geen van hierdie/None of these	

Vraag 6 /Question 6

Beskou die funksies $f(x) = \ln(x)$ en $g(x) = \sqrt{x}$.
Die definisieversameling van die funksie $f^{-1} + g$ is

Consider the functions $f(x) = \ln(x)$ en $g(x) = \sqrt{x}$.
The domain of the function $f^{-1} + g$ is

[6a] R	[6b] $(-\infty, 0)$	[6c] $(0, \infty)$	[6d] $[0, \infty)$	[6e] Geen van hierdie/None of these

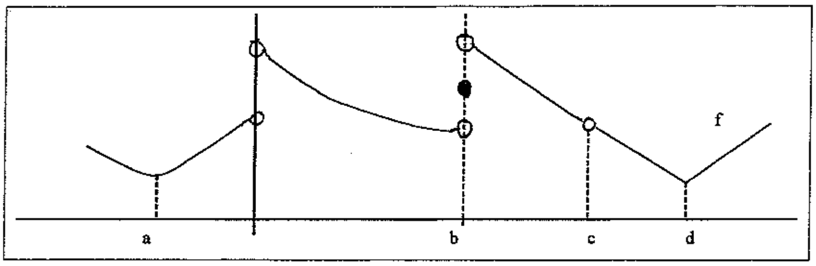
Vraag 7 / Question 7

Indien / If $|x^2+1|<4$, dan is

[7a] $x \in (-\sqrt{3},\sqrt{3}),\ x \neq 0$	[7b] $x < \pm\sqrt{3}$	[7c] $x \in (-\sqrt{3},\sqrt{3})$
[7d] Geen van hierdie/None of these		

Vrae 8 en 9 / Questions 8 and 9

Die grafiek van 'n funksie f word gegee. /The graph of a function f is given.



Vraag 8 / Question 8

Die funksie f is diskontinu slegs by

The function f is discontinuous only at

[8a] $x=0$	[8b] $x=0,\ x=a,\ x=b,\ x=c,\ x=d$	[8c] $x=0,\ x=b,\ x=c$
[8d] $x=0,\ x=c$	[8e] Geen van hierdie/None of these	

Vraag 9 / Question 9

Die funksie f is nie differensieerbaar nie slegs by

The function f is not differentiable only at

[9a] $x=0,\ x=b,\ x=d$	[9b] $x=0,\ x=b,\ x=c,\ x=d$	[9c] $x=d$
[9d] $x=0,\ x=b,\ x=c$	[9e] Geen van hierdie/None of these	

Vrae 10 en 11 / Questions 10 and 11

Beskou die funksies / Consider the functions

$y=\sin\sqrt{x}$, $y=\cos(3x)$, $y=|x-2|$, $y=e^{3x}$ en/and $y=\ln(x-1)$.

Vraag 10 / Question 10

Watter bewering is waar? / Which statement is true?

[10a] Elke funksie is kontinu op R / Each function is continuous on R.
[10b] Elke funksie is kontinu op sy definisieversameling. Each function is continuous on its domain.
[10c] Geen van hierdie/None of these

Vraag 11 / Question 11

Watter bewering is waar? / Which statement is true?

[11a] Elke funksie is differensieerbaar op R / Each function is differentiable on R.
[11b] Elke funksie is differensieerbaar op sy definisieversameling. Each function is differentiable on its domain.
[11c] Geen van hierdie/None of these

Vraag 12 / Question 12

$$\lim_{x\to\infty} \frac{1-\sqrt{x}}{1+\sqrt{x}}$$

[12a] $= -1$	[12b] $= 1$	[12c] $= \infty$	[12d] bestaan nie/does not exist
[12e] Geen van hierdie/None of these			

Vraag 13 / Question 13

Die vertikale asimptote van $f(x) = \dfrac{x^2-1}{(x+5)(x-1)}$ is

The vertical asymptotes of $f(x) = \dfrac{x^2-1}{(x+5)(x-1)}$ are

[13a] $x = -5,\ x = 1$	[13b] $y = -5,\ y = 2$	[13c] $x = 5$	[13d] $x = 1$
[13e] $y = \pm\infty$	[13f] $x = \pm\infty$	[13g] Geen van hierdie/None of these	

[13]

BEANTWOORD ALLE VERDERE VRAE OP HIERDIE VRAESTEL. TOON ALLE BEREKENINGE

ANSWER ALL THE FOLLOWING QUESTIONS ON THIS PAPER AND SHOW ALL COMPUTATIONS CLEARLY

Vraag 14 / Question 14

Los die vergelyking $\sin 2x = -\frac{1}{2},\ x \in [-\pi, \pi]$ op.

Solve the equation $\sin 2x = -\frac{1}{2},\ x \in [-\pi, \pi]$.

reference angle, $\frac{\pi}{6}$

$\sin 2x = -\frac{1}{2} \Rightarrow 2x = (\pi + \frac{\pi}{6}) + 2k\pi = \frac{7\pi}{6} + 2k\pi$

or $2x = -\frac{\pi}{6} + 2k\pi$

$\Rightarrow x = \frac{7\pi}{12} + k\pi$ or $x = -\frac{\pi}{12} + k\pi$, $k \in \mathbb{Z}$

BUT $x \in [-\pi, \pi]$

substitute : $k=0$ in first equation, then $x = \frac{7\pi}{12}$

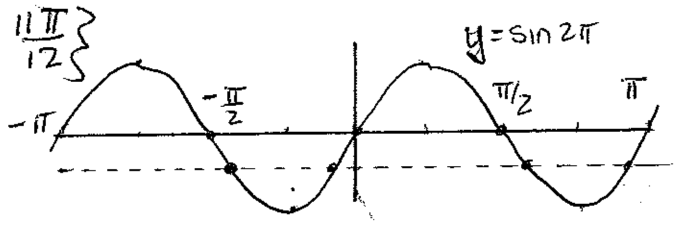
$k=-1$ in first equation, then $x = \frac{7\pi}{12} - \pi = -\frac{5\pi}{12}$

$k=0$ in second equation, then $x = -\frac{\pi}{12}$

$k=1$ in second equation, then $x = -\frac{\pi}{12} + \pi = \frac{11\pi}{12}$

[4]

$\therefore x \in \left\{-\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}\right\}$



Vraag 15 / Question 15

Bepaal k indien $f(x) = e^{kx+2}$ en $f(1) = 3$.
Solve for k if $f(x) = e^{kx+2}$ and $f(1) = 3$.

$$f(1) = 3 \Rightarrow e^{k+2} = 3$$

$$\Rightarrow \ln(e^{k+2}) = \ln 3$$

$$\Rightarrow k+2 = \ln 3$$

$$\Rightarrow k = -2 + \ln 3$$

[2]

Vraag 16 / Question 16

Beskou die funksie / Consider the function

$$f(x) = \begin{cases} \dfrac{3x}{x^2+x} & \text{as/if } x > 0 \\ 3 & \text{as/if } x = 0 \\ \dfrac{\sin 3x}{x} & \text{as/if } x < 0 \end{cases}$$

(i) Bepaal, indien moontlik, $\lim_{x \to -2} f(x)$
Find, if possible, $\lim_{x \to -2} f(x)$.

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{\sin 3x}{x}$$

$$= \frac{\sin(-6)}{-2}$$ (substitution law, quotient law)

[1]

ii Is f kontinu by $x = 0$? **Verduidelik jou antwoord.**
Is f continuous at $x = 0$? **Explain your answer.**

(i) $f(0)=3$

(ii) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{3x}{x^2+x} = \lim\limits_{x\to 0^+} \frac{3x}{x(x+1)}$

$= \lim\limits_{x\to 0^+} \frac{3}{x+1}$

$= \frac{3}{0+1}$ (substitution and quotient limit laws)

$=3$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{\sin 3x}{x} = \lim\limits_{x\to 0^-} 3\,\frac{\sin 3x}{3x}$

$= 3 \lim\limits_{x\to 0^-} \frac{\sin 3x}{3x} = 3\times 1$ (standard limit)

$=3$

as $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = 3$ it follows that

$\lim\limits_{x\to 0} f(x) = 3$ (Theorem)

[5]

(iii) as $f(0) = \lim\limits_{x\to 0} f(x)$ it follows that

f is continuous at $x=0$

Vraag 17 /Question 17

(i) Gebruik eerste beginsels (definisie van afgeleides) om die helling van die raaklyn aan die grafiek van $f(x) = \sqrt{x+2}$ by $x = 0$ te bepaal.
Use first principles (definition of derivative) to find the gradient of the tangent line to the graph of $f(x) = \sqrt{x+2}$ at $x = 0$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \quad \left(\text{vorm } \frac{0}{0}\right)$$

$$= \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h\to 0} \frac{(2+h) - 2}{h(\sqrt{2+h} + \sqrt{2})} = \lim_{h\to 0} \frac{h}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$

gradient is $f'(0) = \frac{1}{2\sqrt{2}}$

[3]

(ii) Gee die vergelyking van die raaklyn in vraag 17(i).
Give the equation of the tangent line in question 17(i).

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{1}{2\sqrt{2}} = \frac{y - \sqrt{2}}{x - 0}$$

$$\Rightarrow y - \sqrt{2} = \frac{1}{\sqrt{2}\,2} x$$

$$\Rightarrow y = \frac{1}{2\sqrt{2}} x + \sqrt{2}$$

[1]

Vraag 18 / Question 18

Bepaal die volgende limiete indien dit bestaan.

Toon alle berekeninge en verduidelik jou antwoorde.

Find the following limits if the exist.

Show all calculations and explain your answers.

(i) $\lim_{x\to\infty} \frac{\sqrt{x}}{x+1}$. (form $\frac{\infty}{\infty}$)

$= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{1+\frac{1}{x}}$

$= \frac{0}{1+0}$ (quotient law and theorem $\lim_{x\to\infty} \frac{k}{x^r} = 0$)

$= 0$

[2]

(ii) $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$. (form $\frac{0}{0}$)

$= \lim_{x\to 2}\left[\frac{2-x}{2x} \times \frac{1}{x-2}\right] = \lim_{x\to 2}\left[\frac{-(x-2)}{2x} \times \frac{1}{x-2}\right]$

$= \lim_{x\to 2} \frac{-1}{2x}$

$= -\frac{1}{4}$ (as $y = -\frac{1}{2x}$ is continuous at $x=2$, you can substitute x with 2)

[2]

(iii) $\lim_{x\to -0.5^-} \frac{|2x+1|}{2x+1}$.

$= \lim_{x\to -0.5^-} \frac{-(2x+1)}{2x+1}$ from definition: $|2x+1| = -(2x+1)$ if $2x+1<0$

$\therefore |2x+1| = -(2x+1)$ if $x < -\frac{1}{2}$

$= \lim_{x\to -0,5^-} -1$

$= -1$ ($y=-1$ is continuous on $\mathbb{R}$ can substitute)

[2]

ROFWERK / SCRIBBLING

As u nog ruimte vir 'n antwoord nodig het, gebruik enige blanko spasie en dui dit duidelik aan **deur 'n raam daarom te trek.**

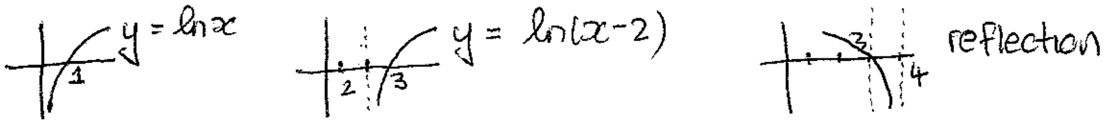
If you need more space for an answer, use any blank space and indicate it clearly **by framing it.**

Question 2

as $f(2)=\sqrt{2}$ and $f(x-1)$ is only a shift to the right,

$2f(x-1) = 2\sqrt{2}$

Question 3



Question 4

$g\circ f(x) = g(x^3+6)$ no f.

Question 5

$$\lim_{x\to 0} \frac{ax^2}{\sin x} = \lim_{x\to 0} a \times \frac{x}{\sin x} \times x = a \times 1 \times 0 = 0$$

Question 6

If $f(x) = \ln(x)$ then $f^{-1}(x) = e^{x}$

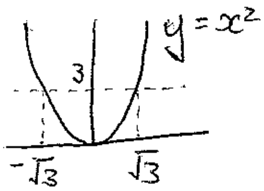
$$D_{f^{-1}+g} = D_{f^{-1}} \cap D_g = \mathbb{R} \cap [0,\infty) = [0,\infty)$$

Question 7

$|x^2+1| < 4 \Rightarrow -4 < x^2+1 < 4$

$\Rightarrow -5 < x^2 < 3$

$\Rightarrow x \in (-\sqrt{3}, \sqrt{3})$



ROFWERK / SCRIBBLING

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Question 9

If the function is not continuous at a point, it is also not differentiable at a point. $x=d$ is also included because the graph makes an angle (sharp corner)

Question 10

A very important theorem

Question 11

The absolute value function $y=|x-2|$ is not differentiable at $x=2$

Question 12

$$\lim_{x\to\infty}\frac{1-\sqrt{x}}{1+\sqrt{x}}=\lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}-1}{\frac{1}{\sqrt{x}}+1}=\frac{0-1}{0+1}=-1$$

Question 13

$$f(x)=\frac{x^2-1}{(x+5)(x-1)}=\frac{(x-1)(x+1)}{(x-1)(x+5)}=\frac{x+1}{x+5}$$

as $\lim_{x\to -5^+} f(x)=\mp\infty$, $x=-5$ is the only vertical asymptote.