WTW158 Memo sem test 1 2009

AFDELING A : MEERVOUDIGE KEUSE VRAE - 14 PUNTE

Gebruik 'n sagte potlood.

Vul jou persoonlike inligting in potlood in op die merkleesvorm se **KANT 2**.

Vul jou studentenommer in van bo na onder en kodeer dit.

Indien jou studentenommer verkeerd gekodeer is of nie ingevul is nie, sal jy nie punte vir afdeling A kry nie.

Beantwoord vrae 1 tot 14 op die MERKLEESVORM se **KANT 2**.

Indien kant 1 gebruik word sal dit nie nagesien word nie.

Omkring jou antwoorde in hierdie vraestel en merk die antwoorde op die vorm as jy seker is van jou keuse.

Jy mag nie verkeerde antwoorde uitvee op die merkleesvorm nie.

SECTION A: MULTIPLE CHOICE QUESTIONS - 14 MARKS

Use a soft pencil.

Fill in your personal information in pencil on the optic reader form on **SIDE 2**.

Fill in your student number from top to bottom and then code it.

If you make a mistake when coding your student number or if the student number is not coded, you will get no marks for section A.

Answer questions 1 to 14 on the OPTIC READER FORM on **SIDE 2**.

If side 1 is used it will not be marked.

First circle your answers in this paper and only mark the answers on the form when you are certain of your choice.

You may not erase wrong answers on the optic reader form.

Vraag 1 / Question 1

Indien $|x-1| \geq 2$, dan is

If $|x-1| \geq 2$, then

[1a] $x \in [3,\infty)$	[1b] $x \in (-\infty,-3] \cup [3,\infty)$	[1c] $x \in [-1,3]$	[1d] $x \in (-\infty,-1] \cup [3,\infty)$
[1e] Geen van hierdie / None of these			

Vraag 2 / Question 2

Die definisieversameling van die funksie $f(x) = \dfrac{2x}{\sqrt{2-x-x^2}} = \dfrac{2x}{\sqrt{(2+x)(1-x)}}$ is

The domain of the function $f(x) = \dfrac{2x}{\sqrt{2-x-x^2}} = \dfrac{2x}{\sqrt{(2+x)(1-x)}}$ is

[2a] $\{x \mid x \neq -2,\ x \neq 1\}$	[2b] $\{x \mid x \neq -2, 0, 1\}$	[2c] $\{x \mid -2 < x < 1\}$
[2d] $(-\infty,-2) \cup (1,\infty)$	[2e] Geen van hierdie / None of these	

The memo for the short questions is at the back (p 11,12)

Vraag 3 / Question 3

$\sin(\frac{5\pi}{3}) =$

[3a] $\frac{\sqrt{3}}{2}$	[3b] $-\frac{\sqrt{3}}{2}$	[3c] $\frac{1}{2}$	[3d] $-\frac{1}{2}$	[3e] $\frac{1}{\sqrt{2}}$	[3f] $-\frac{1}{\sqrt{2}}$
[3g] Geen van hierdie / None of these					

Vraag 4 / Question 4

bgtan$(-\frac{1}{\sqrt{3}}) = \tan^{-1}(-\frac{1}{\sqrt{3}}) =$

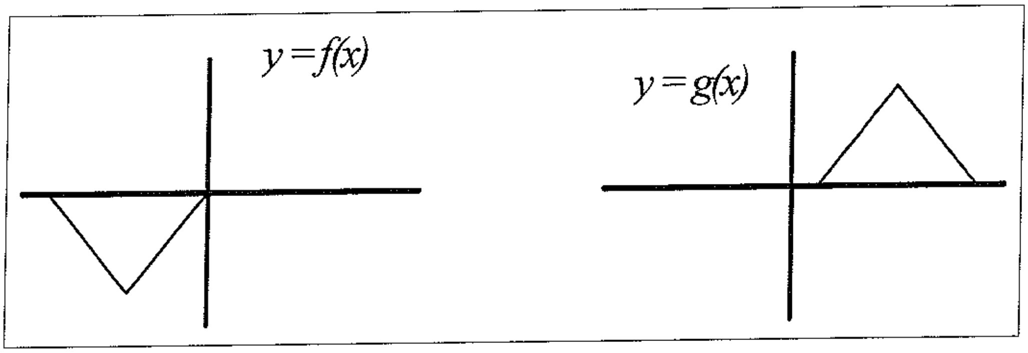
arctan$(-\frac{1}{\sqrt{3}}) = \tan^{-1}(-\frac{1}{\sqrt{3}}) =$

[4a] $\frac{\pi}{3}$	[4b] $\frac{\pi}{6}$	[4c] $-\frac{\pi}{6}$	[4d] $\frac{4\pi}{6}$	[4e] $-\frac{\pi}{3}$	[4f] $\frac{5\pi}{3}$
[4g] Geen van hierdie / None of these					

Vraag 5 / Question 5

Beskou die grafieke van die funksies $y = f(x)$ en $y = g(x)$ hieronder:

Consider the graphs of the functions $y = f(x)$ and $y = g(x)$ below:



$g(x) = \ldots$

[5a] $f(x-a),\ a>0$	[5b] $-f(x-a),\ a>0$	[5c] $f(x+a),\ a>0$
[5d] $-f(x+a),\ a>0$	[5e] $-f(x)-a,\ a>0$	[5f] $f(x)-a,\ a>0$
[5g] Geen van hierdie / None of these		

Vraag 6 / Question 6

Indien $12 - e^{0.4t} = 3$, dan is $t =$

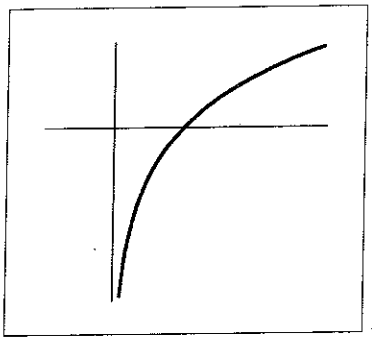
If $12 - e^{0.4t} = 3$, then $t =$

[6a] $\frac{\ln 12 - \ln 3}{0.4}$	[6b] $\frac{\ln 4}{0.4}$	[6c] $\frac{\ln 9}{0.4}$	[6d] Geen van hierdie / None of these

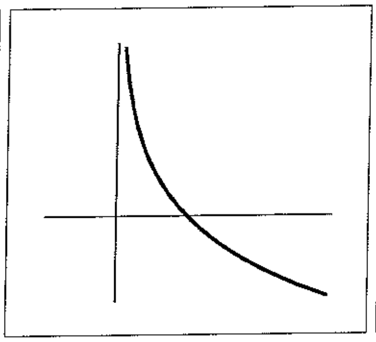
Vraag 7 / Question 7

Die grafiek van die funksie $f(x) = \ln(-x)$ is

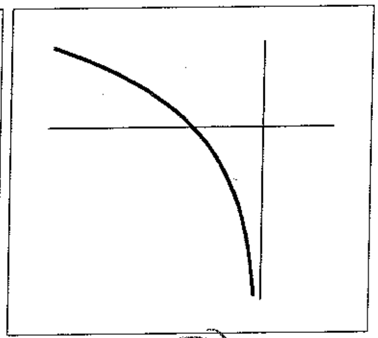
The graph of the function $f(x) = \ln(-x)$ is



[7a]



[7b]



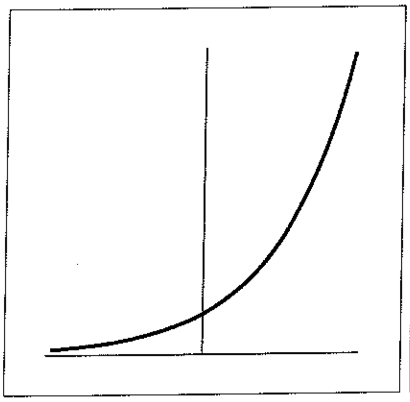
[7c]

[7d] Geen van hierdie / None of these

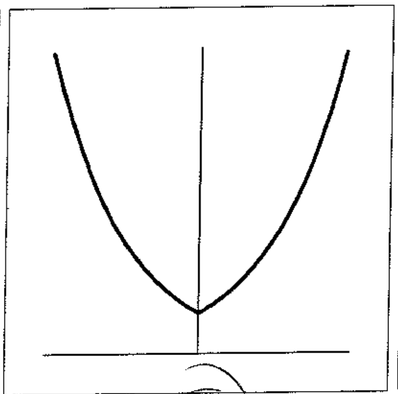
Vraag 8 / Question 8

Die grafiek van die funksie $f(x) = e^{|x|}$ is

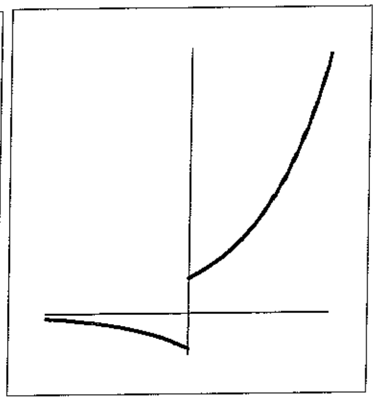
The graph of the function $f(x) = e^{|x|}$ is



[8a]



[8b]



[8c]

[8d] Geen van hierdie / None of these

Vraag 9 / Question 9

bgcos$(\cos(-\frac{\pi}{4})) = \cos^{-1}(\cos(-\frac{\pi}{4})) =$

arccos$(\cos(-\frac{\pi}{4})) = \cos^{-1}(\cos(-\frac{\pi}{4})) =$

[9a] $-\frac{\pi}{4}$	[9b] $\frac{\pi}{4}$	[9c] $\frac{1}{\sqrt{2}}$	[9d] $-\frac{1}{\sqrt{2}}$	[9e] Geen van hierdie / None of these

Vraag 10 / Question 10

Indien $\sin\theta = \frac{2}{\sqrt{13}}$ en $\sec\theta = -\frac{\sqrt{13}}{3}$, dan is $\cot\theta =$

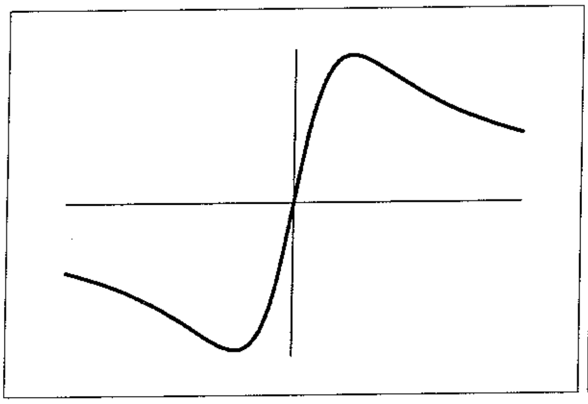
If $\sin\theta = \frac{2}{\sqrt{13}}$ and $\sec\theta = -\frac{\sqrt{13}}{3}$, then $\cot\theta =$

[10a] $\frac{2}{3}$	[10b] $-\frac{2}{3}$	[10c] $\frac{3}{2}$	[10d] $-\frac{3}{2}$	[10e] Geen van hierdie / None of these

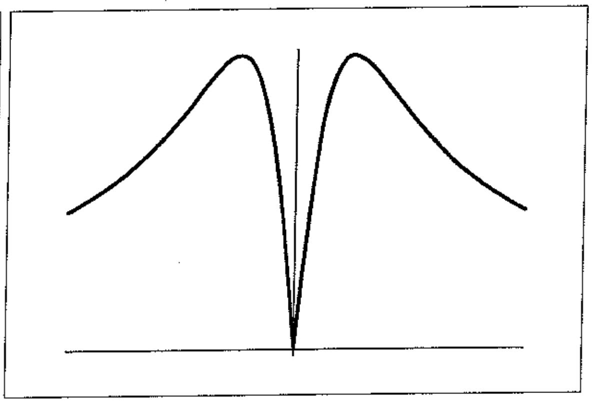
Vraag 11 / Question 11

Beskou die grafieke van die funksies $y = f(x)$ en $y = g(x)$.
Beide funksies het dieselfde definisieversameling **D**.

Consider the graphs of the functions $y = f(x)$ and $y = g(x)$.
Both functions have the same domain **D**



$y = f(x)$



$y = g(x)$

Beskou die bewerings / Consider the statements:

I: $f(x) = |g(x)|$ vir alle / for all $x \in$ **D**

II: $g(x) = |f(x)|$ vir alle / for all $x \in$ **D**

III: $g(x) = |g(x)|$ vir alle / for all $x \in$ **D**

Watter bewering(s) is WAAR?
Which statement(s) is(are) TRUE?

[11a] slegs / only **I**	[11b] slegs / only **II**
[11c] slegs / only **III**	[11d] slegs / only **I** en / and **II**
[11e] slegs / only **I** en / and **III**	[11f] slegs / only **II** en / and **III**
[11g] Geen van hierdie / None of these	

Vraag 12 / Question 12

Indien $f(x) = \dfrac{4x}{|-2x|}$, dan is $\lim\limits_{x \to 0^-} f(x)$

If $f(x) = \dfrac{4x}{|-2x|}$, then $\lim\limits_{x \to 0^-} f(x)$

[12a] $= -2$	[12b] $= 2$	[12c] bestaan nie / does not exists
[12d] $= -1$	[12e] $= 1$	[12f] Geen van hierdie / None of these

Vraag 13 / Question 13

Die funksie $d = f(t)$ hieronder gee die afstand afgelê na t sekondes deur 'n bewegende voorwerp.

Watter bewering is WAAR?

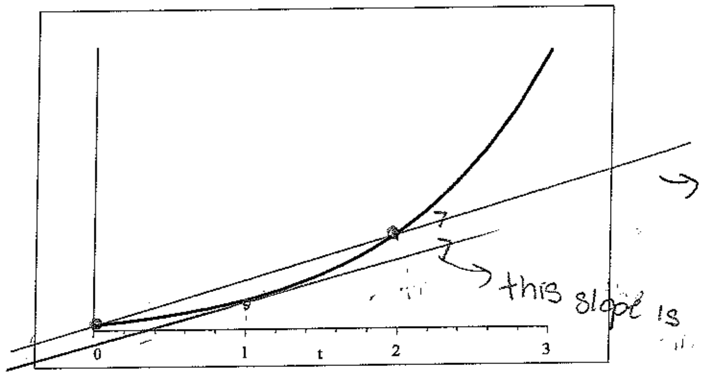
[13a] die gemiddelde snelheid oor die interval $[0,2]$ is minder as die snelheid op tydstip $t = 1$
[13b] die gemiddelde snelheid oor die interval $[0,2]$ is meer as die snelheid op tydstip $t = 1$
[13c] die gemiddelde snelheid oor die interval $[0,2]$ is gelyk aan die snelheid op tydstip $t = 1$
[13d] Geen van hierdie

The function $d = f(t)$ below gives the distance covered after t seconds by a moving object.

Which statement isTRUE.

[13a] the average velocity over the interval $[0,2]$ is less than the velocity at time $t = 1$
[13b] the average velocity over the interval $[0,2]$ is greater than the velocity at time $t = 1$
[13c] the average velocity over the interval $[0,2]$ is equal to the velocity at time $t = 1$
[13d] None of these

lines are parallel.



the slope of this line is the average velocity

this slope is the velocity

Vraag 14 / Question 14

Indien $\dfrac{|x-1|}{|x|-1} < 0$, dan is / If $\dfrac{|x-1|}{|x|-1} < 0$, then

[14a] $x \neq 1$	[14b] $x \in (-1,1)$	[14c] $x > 1$
[14d] $x \in \phi$ (die ongelykheid het geen oplossings nie / the inequality has no solutions)		
[14e] Geen van hierdie/None of these		

[14]

AFDELING B: 26 PUNTE

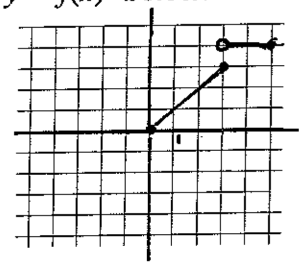
BEANTWOORD ALLE VERDERE VRAE OP HIERDIE VRAESTEL IN INK.

SECTION B: 26 MARKS

ANSWER ALL THE FOLLOWING QUESTIONS ON THIS PAPER IN INK.

Vraag 15 / Question 15

Beskou die grafiek van 'n funksie $y = f(x)$ hieronder.
Consider the graph of a function $y = f(x)$ below.

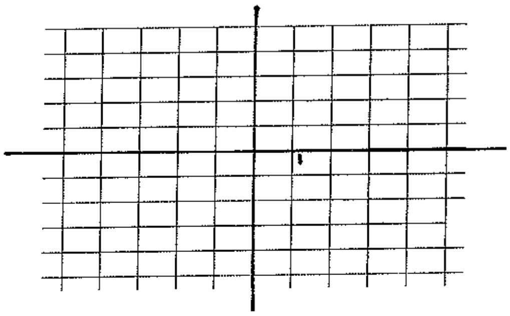


i Skryf die waardeversameling van die funksie neer.
Write down the range of the function.

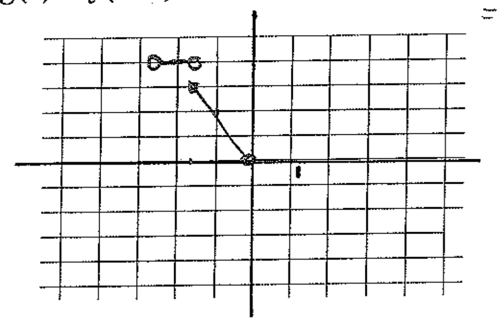
$[0,3] \cup \{4\} = \{x \mid 0 \leq x \leq 3 \text{ or } x = 4\}$ [1]

ii Skets **(in ink)** die grafiek van die funksie $g(x) = f(-2x)$.
Sketch **(in ink)** the graph of the function $g(x) = f(-2x)$.

example $g(-\frac{5}{2})$
$= f(-2 \times -\frac{5}{2}) = f(5)$
$= 4$



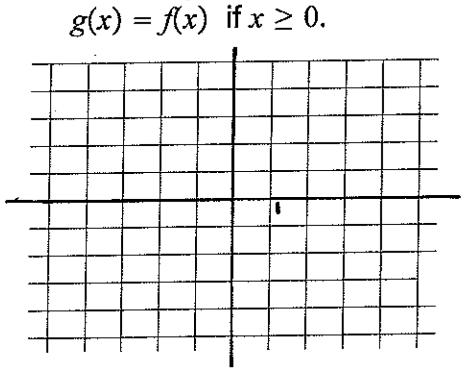
Rofwerk / Rough work



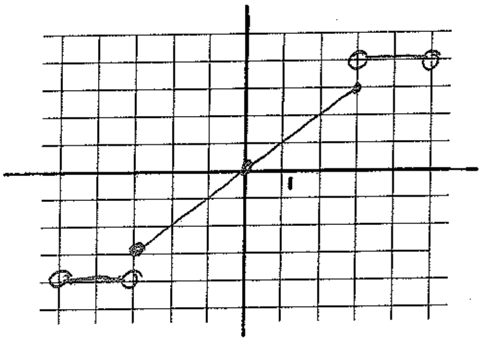
Antwoord / Answer

[2]

iii Skets **(in ink)** 'n funksie $y = g(x)$ sodanig dat g 'n onewe funksie is en $g(x) = f(x)$ as $x \geq 0$.
Sketch **(in ink)** a function $y = g(x)$ such that g is an odd function and $g(x) = f(x)$ if $x \geq 0$.



Rofwerk / Rough work



Antwoord / Answer

[2]

g is odd if $g(-x) = -g(x)$

Vraag 16 / Question 16

Los op $\frac{3\ln x}{1+\ln\sqrt{x}} = 4$. Toon duidelik aan hoe u die eienskappe van die ln- funksie gebruik.

Solve $\frac{3\ln x}{1+\ln\sqrt{x}} = 4$. Clearly show how you use the properties of the ln- function.

$$\frac{3\ln x}{1+\ln\sqrt{x}} = 4 \Rightarrow 3\ln x = 4(1+\ln\sqrt{x}) = 4 + 4\ln\sqrt{x}.$$

$$\Rightarrow \ln x^3 = 4 + \ln(\sqrt{x})^4 = 4 + \ln x^2$$

$$\Rightarrow \ln x^3 - \ln x^2 = 4$$

$$\Rightarrow \ln \frac{x^3}{x^2} = 4$$ ($x \neq 0$ because $\ln x$ is defined if $x > 0$)

$$\Rightarrow \ln x = 4$$

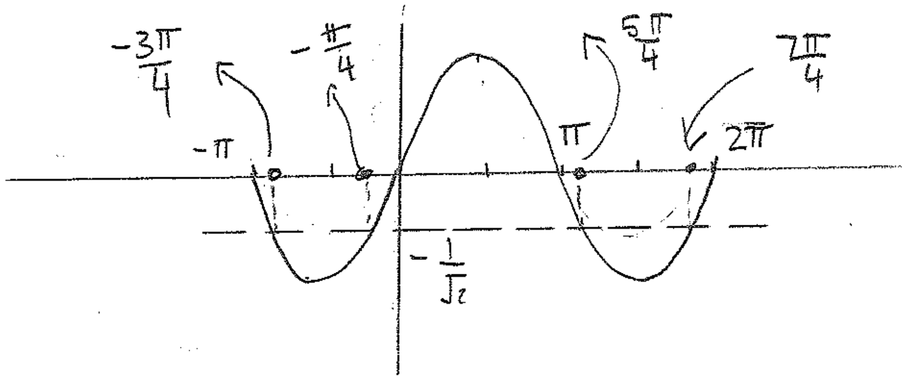
$$\Rightarrow x = e^4$$

[2]

Vraag 17 / Question 17

i Skets die grafiek van $f(x) = \sin x,\ x \in [-\pi, 2\pi]$.

Sketch the graph of $f(x) = \sin x,\ x \in [-\pi, 2\pi]$.



[1]

ii Gebruik die grafiek en los op $\sin x > -\frac{1}{\sqrt{2}},\ x \in [-\pi, 2\pi]$.

Use the graph and solve $\sin x > -\frac{1}{\sqrt{2}},\ x \in [-\pi, 2\pi]$.

$$\sin x = -\frac{1}{\sqrt{2}},\ x \in [-\pi, 2\pi]$$

$$\Rightarrow x = \pi + \frac{\pi}{4} = \frac{5\pi}{4} \text{ or } x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$\text{or } x = -\frac{\pi}{4} \text{ or } x = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$$

see graph

[2]

$$\sin x > -\frac{1}{\sqrt{2}}$$

$$\Rightarrow x \in \left[-\pi, -\frac{3\pi}{4}\right) \cup \left(-\frac{\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$$

Vraag 18 / Question 18

Laat $f(x) = x^2 - 2$ en $g(x) = 2\sin x$. Bepaal $(f \circ g)(x)$ en skryf jou antwoord as 'n veelvoud van een trigonometriese funksie.

Let $f(x) = x^2 - 2$ and $g(x) = 2\sin x$. Find $(f \circ g)(x)$ and write your answer as a multiple of one trigonometric function.

$$(f \circ g)(x) = f(g(x)) = f(2\sin x)$$
$$= (2\sin x)^2 - 2 = 4\sin^2 x - 2$$
$$= 2(2\sin^2 x - 1) = -2(1 - 2\sin^2 x)$$
$$= -2\cos 2x$$

[2]

Vraag 19 / Question 19

Laat / Let $f(x) = \dfrac{\frac{1}{x} - \frac{1}{7}}{7 - x}$.

i Bepaal, indien dit bestaan, $\lim\limits_{x \to 7} f(x)$.

Find, if it exists, $\lim\limits_{x \to 7} f(x)$.

$$\lim_{x \to 7} f(x) = \lim_{x \to 7} \frac{\frac{1}{x} - \frac{1}{7}}{7 - x}$$
$$= \lim_{x \to 7} \frac{\frac{7 - x}{7x}}{7 - x}$$
$$= \lim_{x \to 7} \frac{7 - x}{7x} \times \frac{1}{7 - x}$$
$$= \lim_{x \to 7} \frac{1}{7x}$$
$$= \frac{1}{49}$$

[2]

ii Het die funksie 'n vertikale asimptoot by $x = 7$? Verduidelik jou antwoord.

Does the function has a vertical asymptote at $x = 7$? Explain your answer.

no.

$$\lim_{x \to 7^+} f(x) = \lim_{x \to 7^-} f(x) = \lim_{x \to 7} f(x) = \frac{1}{49}$$

[1]

$x = +7$ can only be a vertical asymptote of any one of the 3 limits above is equal to ∞ or $-\infty$

Vraag 20 / Question 20

Laat / Let $f(x) = \frac{x^2 - 9}{|x - 3|}$.

i Skryf f as 'n stuksgewysgedefinieerde funksie deur van die absolute waarde ontslae te raak. Vereenvoudig u antwoord.

Write f as a piecewise defined function by getting rid of the absolute value. Simplify your answer.

$$f(x) = \frac{x^2-9}{|x-3|} = \begin{cases} \frac{(x-3)(x+3)}{x-3} & \text{if } x-3>0 \\ \frac{(x-3)(x+3)}{-(x-3)} & \text{if } x-3<0 \end{cases}$$

$$= \begin{cases} x+3 & \text{if } x>3 \\ -(x+3) & \text{if } x<3 \end{cases}$$

[2]

ii Bepaal $\lim_{x\to 3} f(x)$, indien dit bestaan.

Find $\lim_{x\to 3} f(x)$, it it exists.

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$$

[2]

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} -(x+3) = -6$$

$\therefore \lim_{x\to 3} f(x)$ does not exists because

$$\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$$

Vraag 21 / Question 21

Bepaal die volgende limiete indien dit bestaan. (Onthou om berekeninge te toon.):

Find the following limits, if the limits do exists. (Remember to show calculations.)

i $\lim\limits_{x \to \pi} \frac{\sin x}{x}$

$$= \frac{\sin \pi}{\pi} = \frac{0}{\pi} = 0$$

ii $\lim\limits_{x \to \pi} \frac{\sin(\pi - x)}{x^2 - \pi^2}$

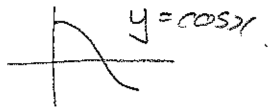
$$= \lim_{x \to \pi} \frac{\sin(\pi - x)}{(x-\pi)(x+\pi)} = \lim_{x \to \pi} \left[\frac{\sin(\pi - x)}{\pi - x} \times \frac{1}{-(x+\pi)} \right]$$

$$= \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x} \times \lim_{x \to \pi} \frac{1}{-(x+\pi)} = 1 \times \frac{1}{-2\pi} = -\frac{1}{2\pi}$$

iii $\lim\limits_{x \to \pi^+} \frac{\cos x}{x - \pi}$

$x \to \pi^+ \Rightarrow x > \pi$

$\Rightarrow x - \pi > 0$



$x \to \pi^- \Rightarrow \cos x \to -1 < 0$

$$= -\infty$$

iv $\lim\limits_{x \to 4^-} \frac{x(\sqrt{x} - 2)}{x - 4}$

$$= \lim_{x \to 4^-} \frac{x(\sqrt{x} - 2)}{x - 4} \times \frac{\sqrt{x} + 2}{\sqrt{x} + 2}$$

$$= \lim_{x \to 4^-} \frac{x(x-4)}{(x-4)(\sqrt{x}+2)} = \lim_{x \to 4^-} \frac{x}{\sqrt{x} + 2}$$

$$= \frac{4}{\sqrt{4} + 2} = \frac{4}{2+2} = \frac{4}{4} = 1$$

v $\lim\limits_{x \to -2^-} \frac{x}{(x+2)^2}$

$x \to -2^- \Rightarrow x < 0$

$x \to -2^- \Rightarrow (x+2)^2 > 0$ [7]

$$= -\infty$$

memo, short questions

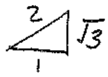
Question 1

$|x-1| \geq 2 \Rightarrow x-1 \geq 2$ or $x-1 \leq -2 \Rightarrow x \geq 3$ or $x \leq -1$

Question 2

$(2+x)(1-x) > 0 \Rightarrow x \in (-2, 1)$

Question 3



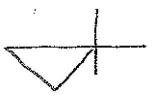
$\sin\left(\frac{5\pi}{3}\right) = -\sqrt{3}/2$

Question 4

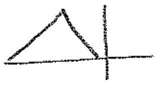
$y = \arctan\left(-\frac{1}{\sqrt{3}}\right) \Leftrightarrow \tan y = -\frac{1}{\sqrt{3}}$ and $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow y = -\pi/6.$

Question 5



$y = f(x)$



$y = -f(x)$

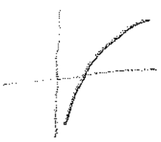
$y = -f(x-a)$

$a > 0$

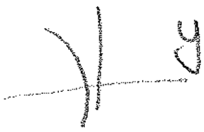
Question 6

$12 - e^{0.4t} = 3 \Rightarrow e^{0.4t} = 9 \Rightarrow 0.4t = \ln 9 \Rightarrow t = \frac{\ln 9}{0.4}$

Question 7



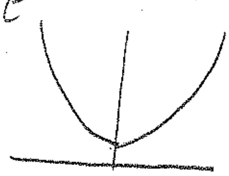
$y = \ln x$



$y = \ln(-x)$

Question 8

$$f(x) = e^{|x|} = \begin{cases} e^{x} & \text{if } x \geq 0 \\ e^{-x} & \text{if } x < 0 \end{cases}$$



Question 9

$\arccos(\cos(-\frac{\pi}{4})) = \arccos(+\frac{1}{\sqrt{2}}) = y$

$\Rightarrow \cos y = +\frac{1}{\sqrt{2}}$ and $y \in [0, \pi]$

$\Rightarrow y = \frac{\pi}{4}$

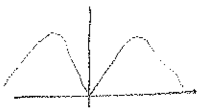
Question 10

$\sin\theta = \frac{2}{\sqrt{13}}$, $\quad \sec\theta = -\frac{\sqrt{13}}{3} \Rightarrow \frac{1}{\cos\theta} = -\frac{\sqrt{13}}{3} \Rightarrow \cos\theta = \frac{-3}{\sqrt{13}}$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{-3}{\sqrt{13}}}{\frac{2}{\sqrt{13}}} = -\frac{3}{\sqrt{13}} \times \frac{\sqrt{13}}{2} = -\frac{3}{2}$$

Question 11

I false, becaus $|g(x)|$ is pos for all x and $f(x) < 0$ if $x < 0$

II true: $|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$  $= g(x)$

III true, $g(x) \geq 0$ therefore $|g(x)| = g(x)$ for $x \in \mathbb{R}$

Question 12

$$f(x) = \frac{4x}{|-2x|} = \begin{cases} \frac{4x}{-2x} & \text{if } -2x > 0 \\ \frac{4x}{-(-2x)} & \text{if } -2x < 0 \end{cases} = \begin{cases} -2 & \text{if } x < 0 \\ 2 & \text{if } x > 0 \end{cases}$$

OR: $f(x) = \frac{4x}{|-2x|} = \frac{4x}{|2x|} = \begin{cases} \frac{4x}{2x} & \text{if } 2x > 0 & = 2 \text{ if } x > 0 \\ \frac{4x}{-2x} & \text{if } 2x < 0 & -2 \text{ if } x < 0 \end{cases}$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -2 = -2$$

Question 13
(see sketch)

Question 14

$\frac{|x-1|}{|x|-1} < 0 \Rightarrow x \neq 1$ and $|x| - 1 < 0 \Rightarrow x \neq 1$ and $|x| < 1$

$\Rightarrow -1 < x < 1$