

EXERCISE 5

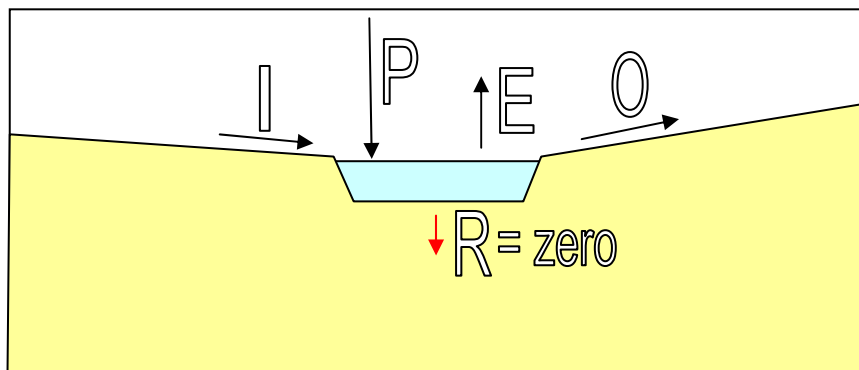
The following data were measured at a water reservoir (5 km² water surface area, impermeable layer/ no leakage at the bottom):

Precipitation rate	4 mm/24 hrs
Change in water level of reservoir due to evaporation	- 1.9 mm
Inflow (tributary river) into the reservoir	100 L/sec
Outflow from the reservoir	40 L/sec

- Schematically depict the water budget.
- Formulate the water balance for the lake.
- Calculate the evaporation losses [mm] during 24 hrs.
- Calculate the total volume of evaporated water.

ANSWER:

a)



b) $\text{Influx} - \text{Outflux} = dV/dt = V_P + V_{\text{in-Out}} - V_{ET} - V_{\Delta wl}$

c) Volume of precipitation (**Note: 1 mm = 1 L/m² = 10³ m³/km²**)
 $V_P = 4 \text{ mm} * 5 \text{ km}^2 = 4 * 10^3 \text{ m}^3/\text{km}^2 * 5 \text{ km}^2 = 20000 \text{ m}^3$

Inflow – outflow = 100 L/s – 40 L/s = 60 L/s = 60 L/s * 10⁻³ m³/L = 0.06 m³/s
 In 24 hrs: $V_{\text{in-out}} = 0.06 \text{ m}^3/\text{s} * 24 \text{ hrs} (60*60 \text{ s/hrs}) = 5184 \text{ m}^3$

d) Volume of water evaporated :
 $\Delta w_l = -1.9 \text{ mm} * 5 \text{ km}^2 = -1.9 * 10^3 \text{ m}^3/\text{km}^2 * 5 \text{ km}^2 = -9500 \text{ m}^3$

$$V_{ET} = V_P + V_{\text{in-Out}} - V_{\Delta w_l}$$

$$V_{ET} = 20000 \text{ m}^3 + 5184 \text{ m}^3 - (-9500) \text{ m}^3 = 34684 \text{ m}^3$$

$$ET = V_{ET} / \text{Area} = 34684 \text{ m}^3 / 5 \text{ km}^2 = 34684 \text{ m}^3 / (5 \text{ km}^2 * 10^6 \text{ m}^2/\text{km}^2)$$

$$ET = 6.937 * 10^{-3} \text{ m} = 6.937 \text{ mm}$$

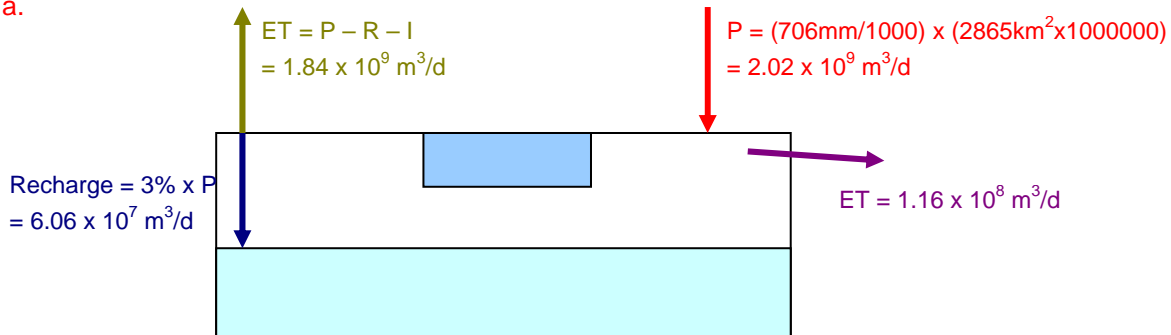
EXERCISE 6

The De Hoop Dam is presently being constructed in the Steelpoort River between Roossenekal and Steelpoort. The catchment covers an area of 2 865 km² with mean annual precipitation of 706 mm and mean annual runoff of 116 million m³. Assuming recharge at 3.0% of MAP, answer the following questions:

- Schematically depict the water budget and balance the inflows and outflows (calculate and show all volumes in m^3/a).
- Calculate the actual annual evaporation in millimetres.
- 80 million m^3 of water from the dam is to be used to supply water to the proximate properties. What – in your opinion – will be the influence of this on the water balance?

ANSWER:

a.



$$\text{Precipitation} = \text{Evapotranspiration} + \text{Runoff} + \text{Recharge}$$

- $ET = ((1.84 \times 10^9 \text{ m}^3/\text{a}) \times 10^{-3}) / 2865 \text{ km}^2 = 643 \text{ mm}$
- Removing water from dam \rightarrow probably removal from surface runoff; i.e.
 $116\,000\,000 - 80\,000\,000 = 36\,000\,000 \text{ m}^3/\text{a}$.

EXERCISE 6

Water is flowing from a water tower into piping of different cross sectional areas, before it leaves the system at point 7.

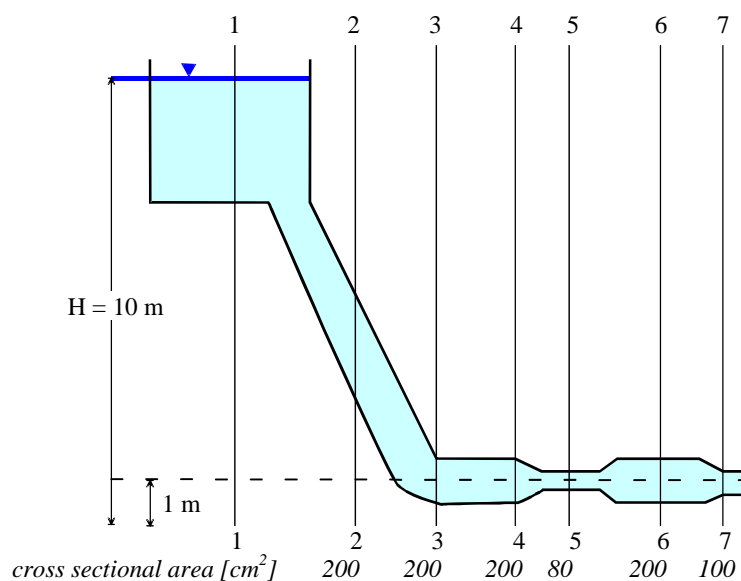
Cross sectional areas of pipes:

$$F_2 = F_3 = F_4 = F_6 = 200 \text{ cm}^2$$

$$F_5 = 80 \text{ cm}^2$$

$$F_7 = 100 \text{ cm}^2$$

What is the hydraulic head, the flow velocity, the pressure head and the elevation head at each point of the pipe network? (Hint: Use the Bernoulli and the continuity equation)



ANSWER:

$$H = \frac{v^2}{2g} + z + \frac{p}{\rho g}$$

Ideal, frictionless fluid \rightarrow no energy lost!

$\Rightarrow H_1 = H_2 = \dots = H_7 = \text{const} = 10 \text{ m}$ (initial head at point 1 = total head)

Position 7: Water leaves pipeline \rightarrow pressure head drops to zero: $\frac{p}{\rho g} = 0$,

elevation head = 1 m:

$$H_7 = \frac{v_7^2}{2g} + z_7 + 0$$

$$v_7 = \sqrt{2g(H_{7(=1)} - z_7)}$$

$$v_7 = 13,28 \text{ m/s} \Rightarrow Q = v_7 F_7 = 0,1328 \text{ m}^3/\text{s}$$

Continuity equation:

$$\Rightarrow Q = v_1 F_1 = v_2 F_2 = v_3 F_3 = v_{\dots} F_{\dots} = v_7 F_7$$

$$\Rightarrow v_2 F_2 = v_7 F_7$$

$$\Rightarrow v_2 = v_7 F_7 / F_2 = 6,65 \text{ m/s}$$

$$\text{Because: } F_2 = F_3 = F_4 = F_6 \Rightarrow v_2 = v_3 = v_4 = v_6 = 6,65 \text{ m/s}$$

$$v_5 = v_7 F_7 / F_5 = 16,60 \text{ m/s}$$

c) pressure head = total head – velocity head – elevation head

$$h = H - \frac{v^2}{2g} - z$$

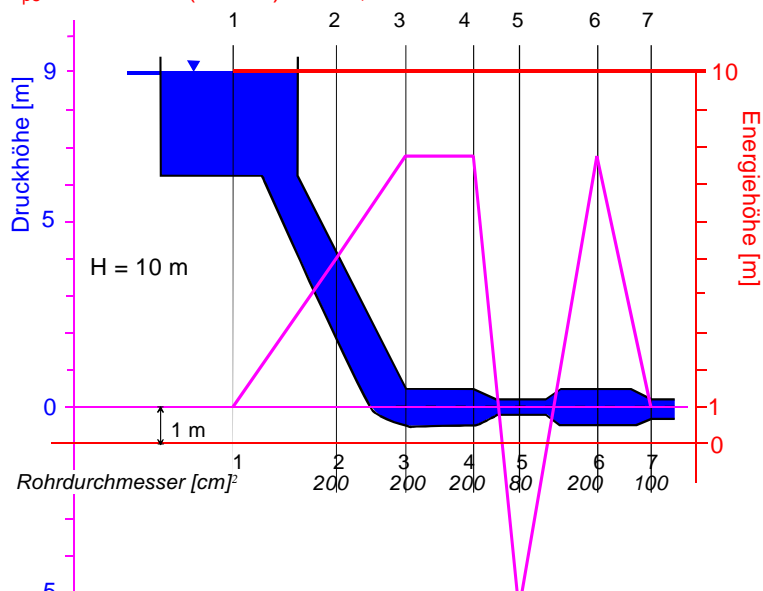
$$h_{p1} = 10 - 0 - 10 = 0$$

$$h_{p7} = 0$$

$$h_{p2} = 10 - 6,65^2 / (2 \cdot 9,81) - 5 = 2,75 \text{ m}$$

$$h_{p3}, h_{p4}, h_{p6} = 10 - 6,65^2 / (2 \cdot 9,81) - 1 = 6,75 \text{ m}$$

$$h_{p5} = 10 - 16,6^2 / (2 \cdot 9,81) - 1 = -5,04 \text{ m}$$



EXERCISE 7

The following parameters were measured in a constant head permeameter test:

$$\Delta h = -5 \text{ cm}$$

$$\Delta l = 10 \text{ cm}$$

Cross sectional area $A = 100 \text{ cm}^2$

Duration of experiment $t = 1 \text{ h}$

Volume of percolated water after 1 h: $V = 3,6 \text{ l}$

a) What is the hydraulic conductivity of the sample?

b) What is the darcy velocity and the linear velocity, if the sample (porosity 0.2) is representative for an aquifer (hydraulic gradient $dh/dL = -0.0025$)?

ANSWER:

$$Q = k_f I A \Rightarrow k_f = Q/(I \cdot A)$$

$$Q = 3.6 \text{ l in } 1 \text{ h} = \frac{3.6 \cdot 10^{-3} \text{ m}^3}{60 \cdot 60 \text{ s}} = \underline{1 \cdot 10^{-6} \text{ m}^3/\text{s}}$$

$$A = 100 \text{ cm}^2 = 0.01 \text{ m}^2$$

$$I = \frac{Dh}{DL} = \frac{5 \text{ cm}}{10 \text{ cm}} = 0.5$$

$$k_f = \frac{1 \cdot 10^{-6} \text{ m}^3/\text{s}}{0.5 \cdot 0.01 \text{ m}^2} = 2 \cdot 10^{-4} \text{ m/s} \rightarrow \text{sand}$$

Darcy velocity

$$v_f = \frac{Q}{F} \Rightarrow v_f = k_f \cdot I \Rightarrow v_f = 2 \cdot 10^{-4} \text{ m/s} \cdot 0.0025 = 5.0 \cdot 10^{-7} \text{ m/s}$$

Linear velocity:

$$v_n = v_a = \frac{v_f}{n_e} = 2.5 \cdot 10^{-6} \text{ m/s}$$